18.06 Goals

The goals for 18.06 are *using matrices and also understanding them.* Here are key computations and some of the ideas behind them:

1. Solving $Ax = b$ for square systems by elimination (pivots, multipliers, back substitution, invertibility of A, factorization into $A = LU$).

2. Complete solution to $Ax = b$ for general A (column space containing b, rank of A, nullspace of A and special solutions to $Ax = 0$ from row reduced R).

3. Basis and dimension (bases for the four fundamental subspaces).

4. Least squares solutions (closest line by understanding projections).

5. Orthogonalization by Gram-Schmidt (factorization into $A = QR$).

6. Properties of determinants (leading to the cofactor formula and the sum over all $n!$ permutations, applications to A^{-1} and volume).

7. Eigenvalues and eigenvectors (diagonalizing A, computing powers A^k and matrix exponentials to solve difference and differential equations).

8. Symmetric matrices and positive definite matrices (real eigenvalues and orthogonal eigenvectors, tests for $x'Ax > 0$, applications).

9. Linear transformations and change of basis (connected to the Singular Value Decomposition—orthonormal bases that diagonalize A).

10. Linear algebra in engineering (graphs and networks, Markov matrices, Fourier matrix, Fast Fourier Transform, linear programming).