Achieving Fast Fully Homomorphic Encryption with Graph Reductions

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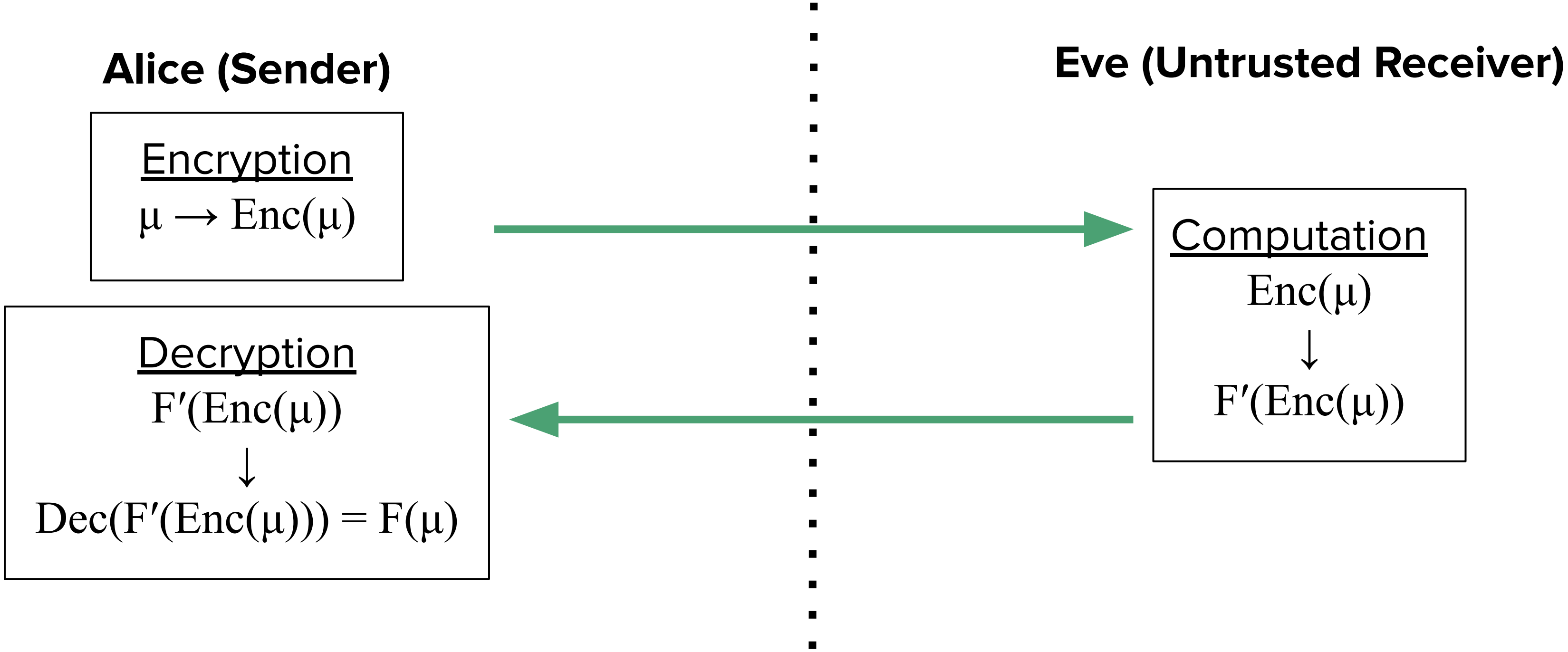
What is fully homomorphic encryption?

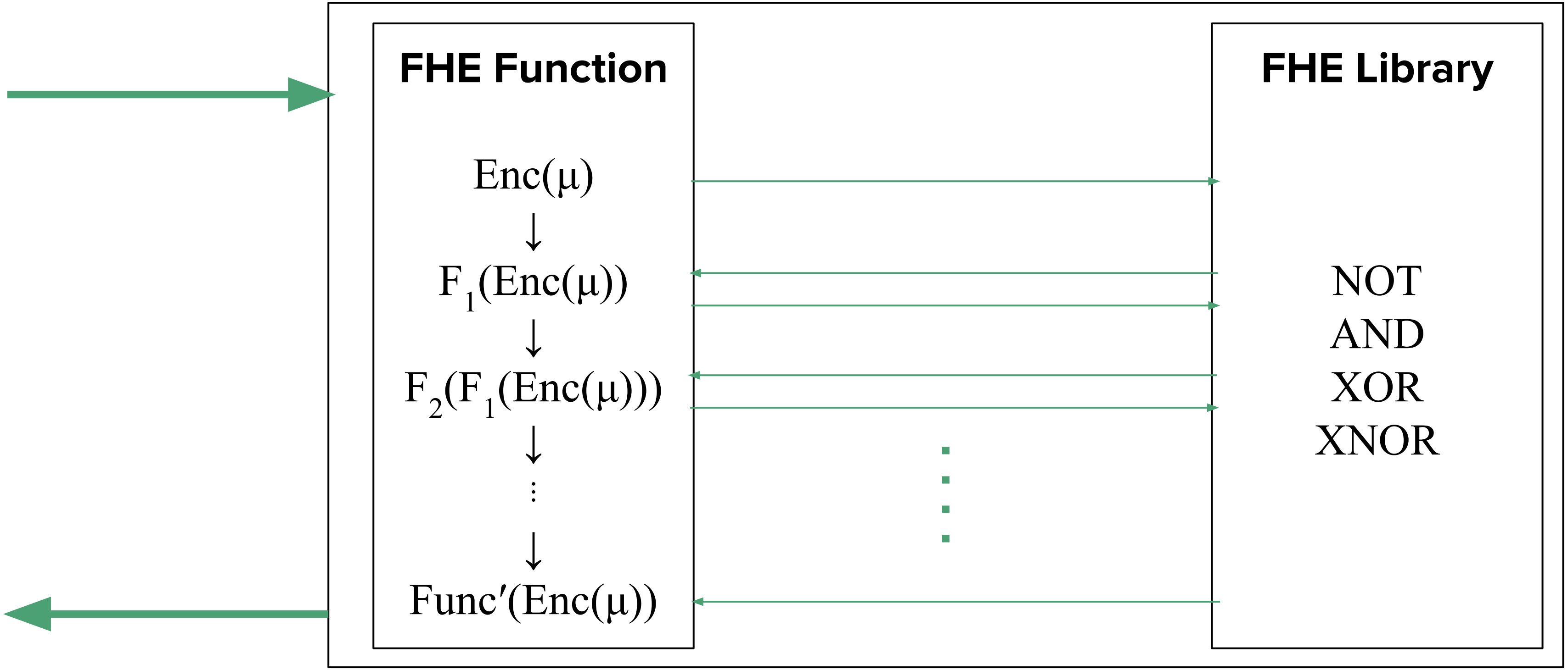
- Support arbitrary computation on encrypted data



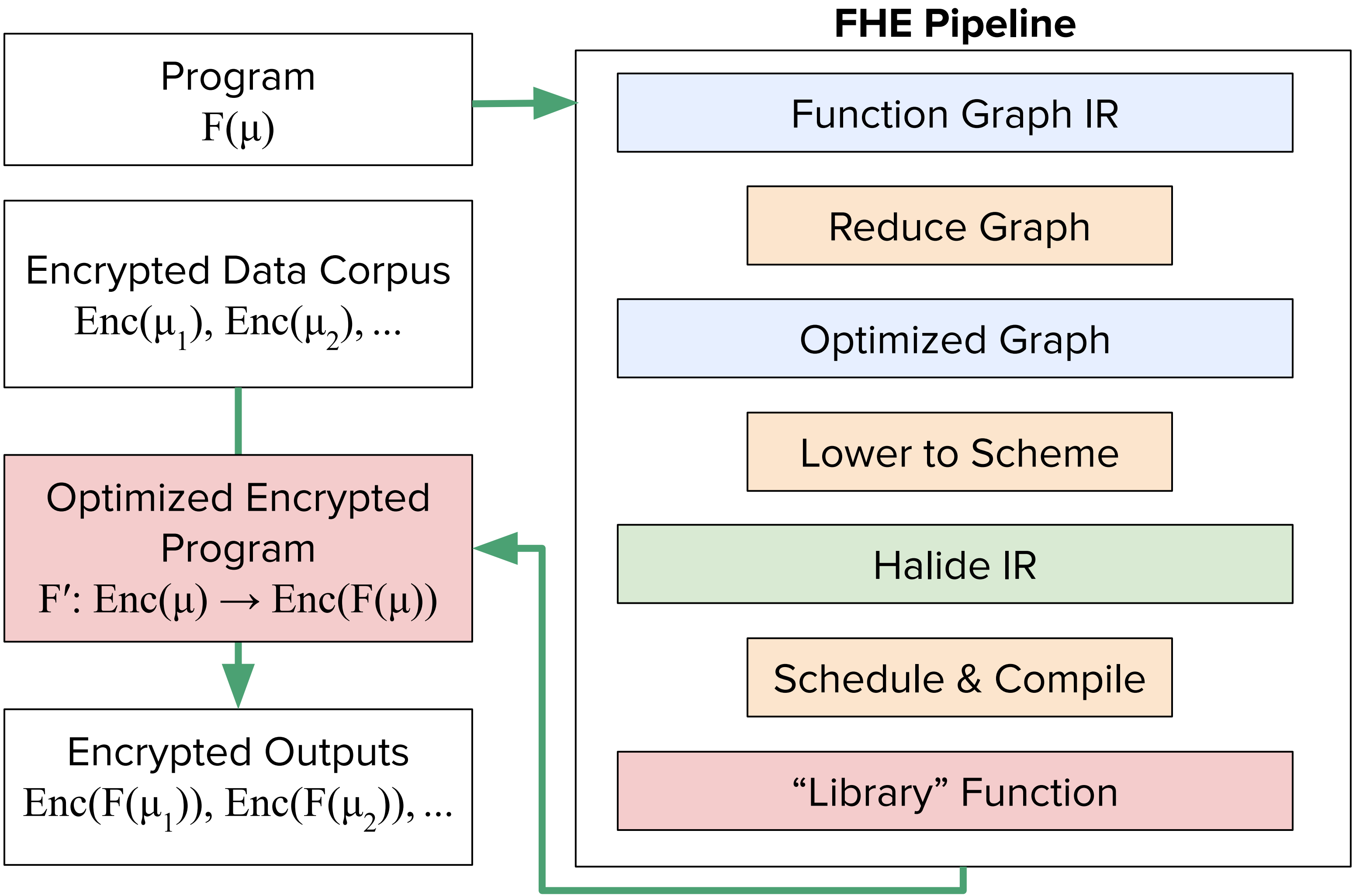
Potential Applications

- We can send tasks off to someone with a more powerful computer or a better algorithm without having to worry about data leaks
  - Filtering email and messages
  - Processing medical data
  - Processing financial data
  - National security

But it is slow



Our Contribution

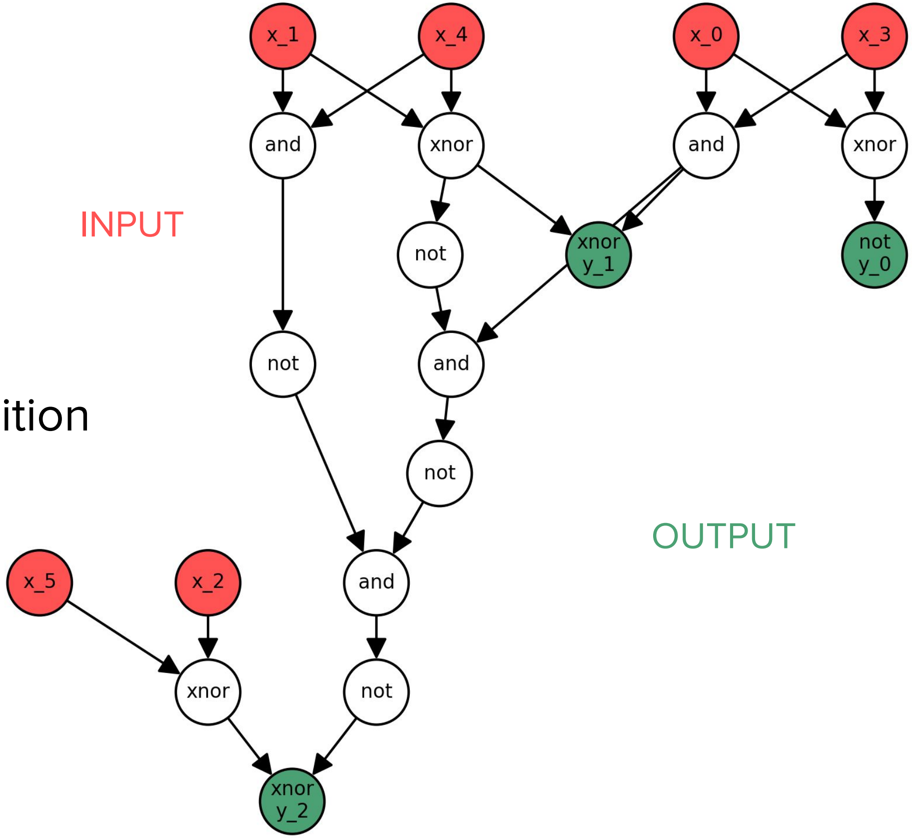


FHE Pipeline
Program
$F(\mu)$
Encrypted Data Corpus
$Enc(\mu_1), Enc(\mu_2), \dots$
Optimized Encrypted Program
$F': Enc(\mu) \rightarrow Enc(F(\mu))$
Encrypted Outputs
$Enc(F(\mu_1)), Enc(F(\mu_2)), \dots$
Function Graph IR
Reduce Graph
Optimized Graph
Lower to Scheme
Halide IR
Schedule & Compile
“Library” Function

Function Graph IR

Function Graphs

- DAG of binary operations



3-bit addition

Measuring Graph Efficiency

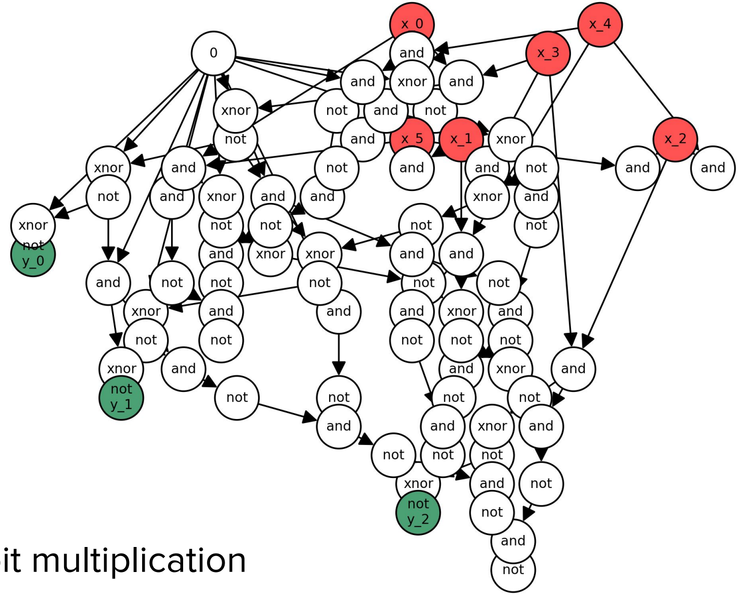
- Benchmark individual binary operations in the FHE scheme

- In the worst case, the time it takes to run the graph is the sum of the time it takes to run each individual operation
    - Could be faster due to parallelism or schedule optimizations

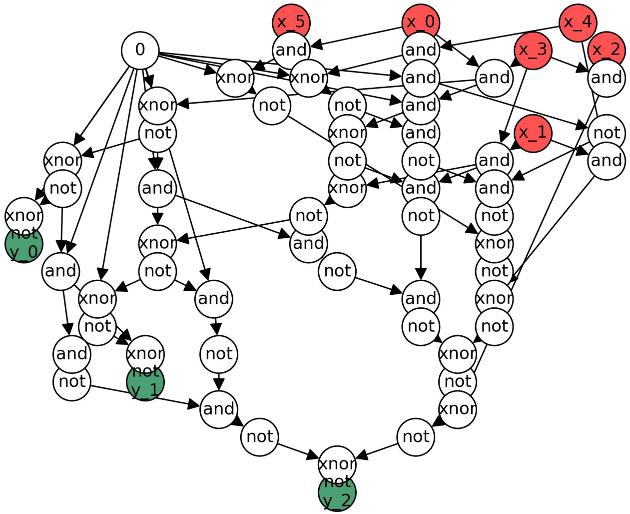
- Theoretically, any scheme can be used

Operation	NOT	AND	XOR	XNOR
Runtime (relative to NOT)	1	18.75	38.71	35.72

Graph Reductions



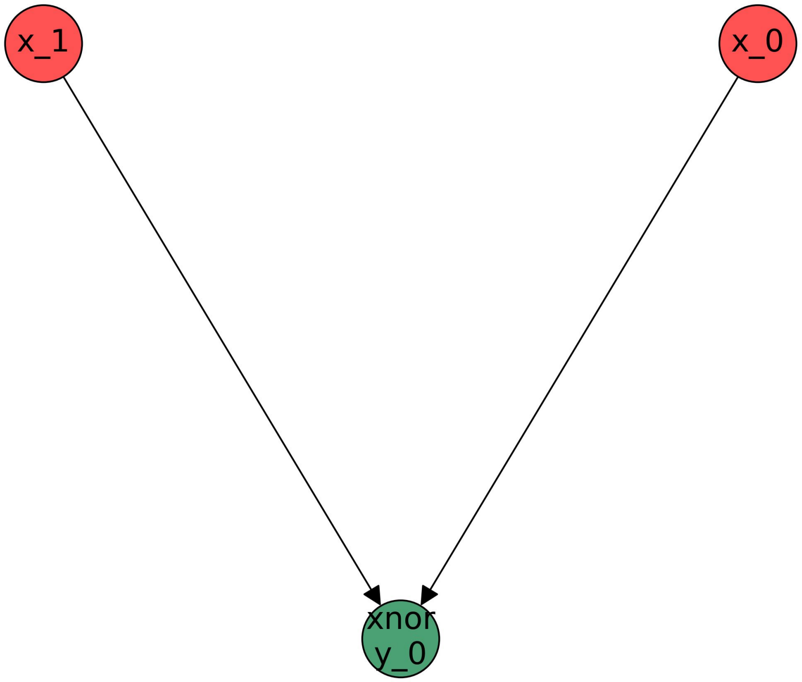
3-bit multiplication



3-bit multiplication
(reduced)

Eliminating Constants and Double NOTs

- Any binary operation taking a constant as an input can be expressed soley in terms of the other input
    - `XOR(A, 1) == NOT(A)`
- NOT(NOT(A)) = A



Optimizing 2-Input Graphs

- Given a graph with two input nodes and some desired outputs, find the best graph to compute those outputs

- 2 inputs ⇒ 4 possible sets of inputs ⇒ 16 possible functions ⇒ 65536 unique sets of outputs
- Run a DP algorithm to find all the optimal graphs and cache them in a table
- Use the table to find the optimal graph for any situation

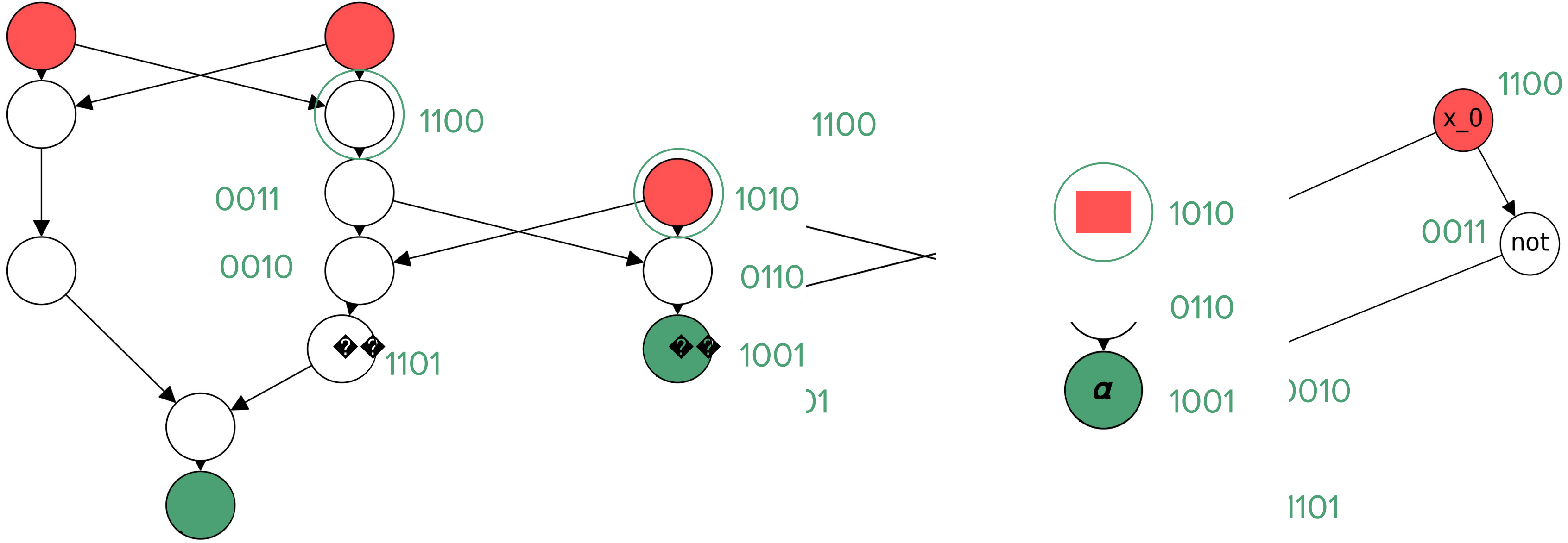
Generic Graph Reduction

For all pairs of nodes u and v:

- Define the subgraph S as all nodes that can be calculated from only u and v
  - Approximate with DFS
- Consider node w in S *interesting* if w is used outside of S or if w is an output of the original graph
- Run the 2-input graph algorithm with interesting nodes as desired outputs
- Replace the S with the ideal subgraph

Repeat until graph cannot be reduced further

Two-Node Reduction on Full Adder



Additional Reduction Methods

- Three-node reduction
- Find exact subgraph $\mathbb{S}$ by running every possible set of inputs and analyzing patterns in node values
- Flag “important” input nodes (ex. sign bits)
    - Try creating separate graphs for when the bit is 0 and when it is 1, then combine with MUX

Scheduling and Compiling

Our FHE Scheme

- GSW 2013: leveled fully homomorphic encryption scheme based on LWE [1]
  - Ciphers are matrices, operations are matrix addition & multiplication
  - Requirement for leveled FHE: plaintext $\mu \in \{0,1\}$ at all times
- NOT (μ) = 1 - μ
- AND (μ_1 μ_2) = μ_1 * u_2
- XOR (μ_1 μ_2) = AND (μ_1 (!μ_2)) + AND ((!μ_1) μ_2)
- XNOR (μ_1 μ_2) = AND (μ_1 μ_2) + AND ((!μ_1) (!μ_2))
  - Graph optimizations take differing costs of operations into account
- Since all encrypted gates are matrix operations, we can use a tensor processing compiler to generate fast code

[1] Gentry, Craig, Sahai, Amit, Waters, Brent 2013

Implementing Fast FHE Operations

- We use Halide, a high-performance image and tensor processing compiler
- Algorithms are separated from schedules
  - Implement FHE operator once
  - Halide can schedule/compile for many architectures (caching differences, CPU/GPU, etc)
- Easy parallelization by design (no side effects, etc)

Homomorphic AND in Halide

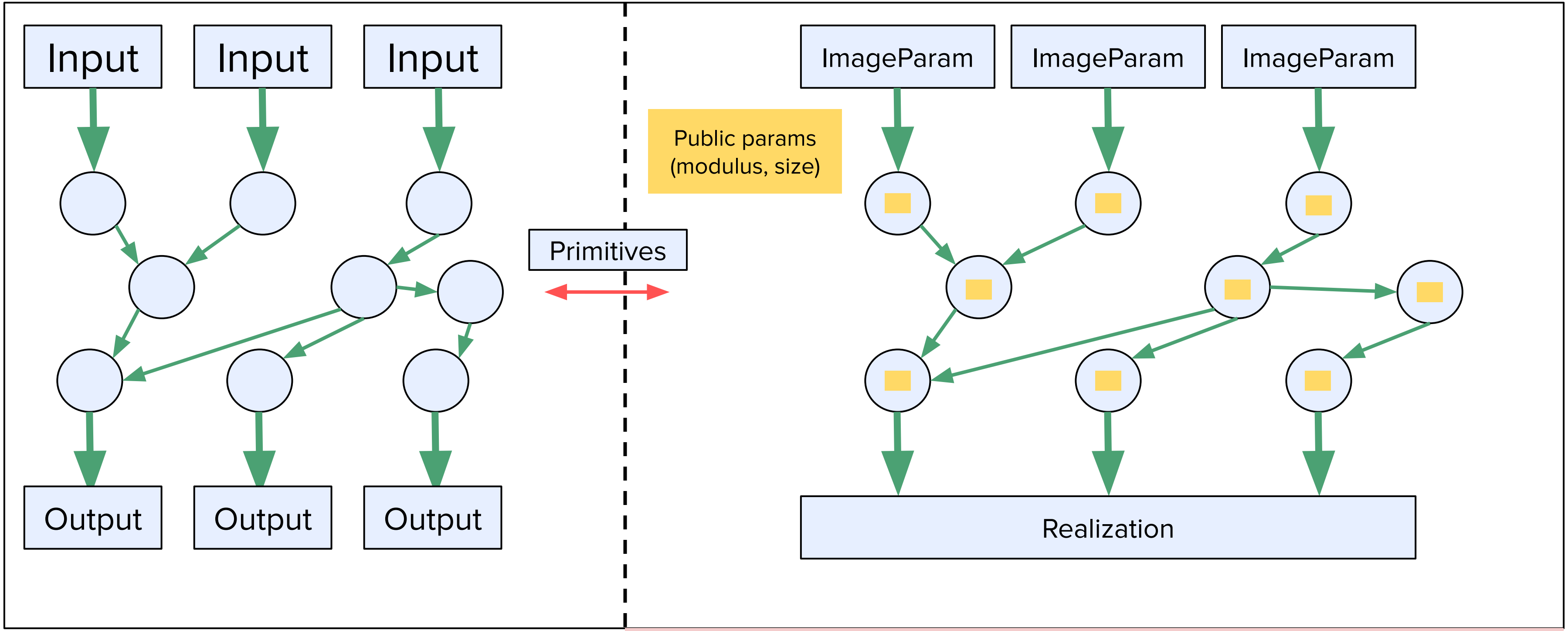
```
//Simplified for ease

Halide::Func AND(Halide::Func f1, Halide::Func f2, int matSize) {
    Halide::Var x, y;
    Halide::RDom r(0, matSize);
    Halide::Func multiply_temp;

    multiply_temp(x, y) = Halide::Expr((int64_t)0);
    multiply_temp(x, y) += f1(x, r) * f2(r, y); //modular sum in practice

    return Flatten(multiply_temp);
}
```

How We Generate Pipelines



Halide compiles this to return a callable function pointer

Compiling a function graph

```
vector<ImageParam> inputPlaceholders(2 * num_bits);

for (int i = 0; i < inputPlaceholders.size(); i++) {
    inputPlaceholders[i] = ImageParam(Int(64), 2);
}

Pipeline hpipe = pipelineGen(some_function, inputPlaceholders, N, q); // pipeline
ready to be scheduled

// scheduling here, or use the auto-scheduler

hpipe.compile_jit(); // or compile_to_c or any other supported language
Realization rel = hpipe.realize(N, N, Target(), params); // ready to be decrypted
```

A “Dynamic” Library

- Given an FHE program, see if we’ve already compiled it, if so return/call it
- Otherwise compile a pipeline to compute the operation
    - Moderately slow, but can be reused
- Can either JIT or ahead-of-time compile depending on use case

API

Creating Graphs: Building From Scratch

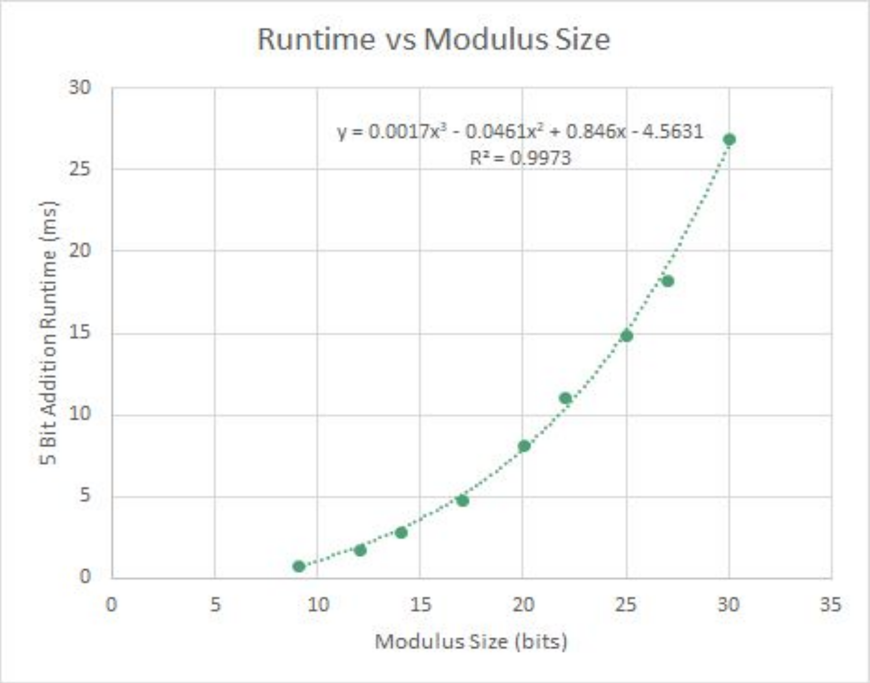
```
function_graph fg(3); // 3 input bits
int node1 = fg.addNode(fg.getInput(0), fg.getInput(1), AND_OP);
int node2 = fg.addNode(fg.getInput(2), node1, OR_OP);
fg.defineOutput(0, node2);
reduce(fg); // also has optional flags
```

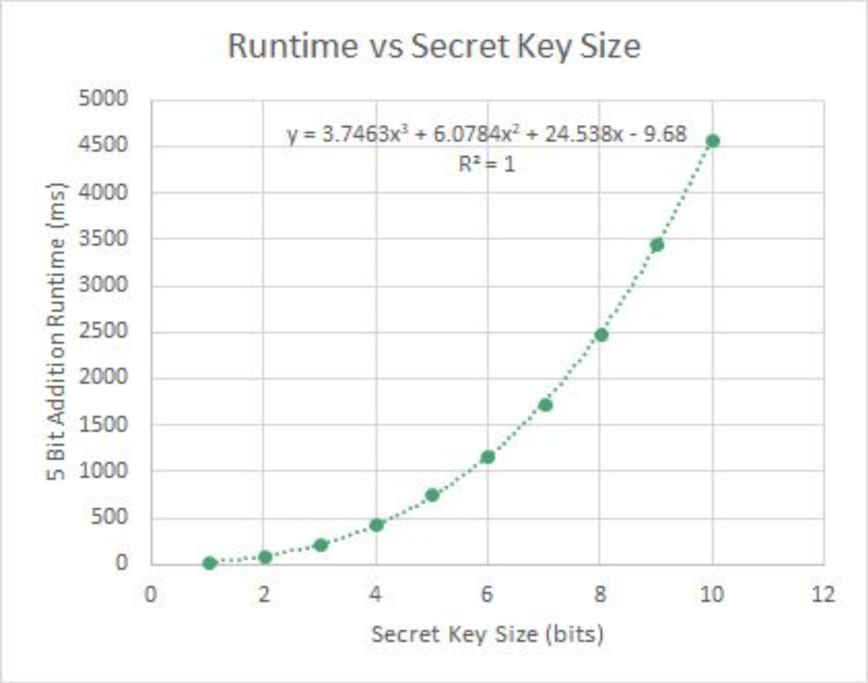
Creating Graphs: Using Standard Operations

```
function_graph fg;
var x(fg, 0, 5); // inputs 0...4
var y(fg, 5, 5); // inputs 5...9
var z(fg, 10, 5); // inputs 10...15
var res = (x + y) / z;
function_graph opGraph = res.realize();
```

Results

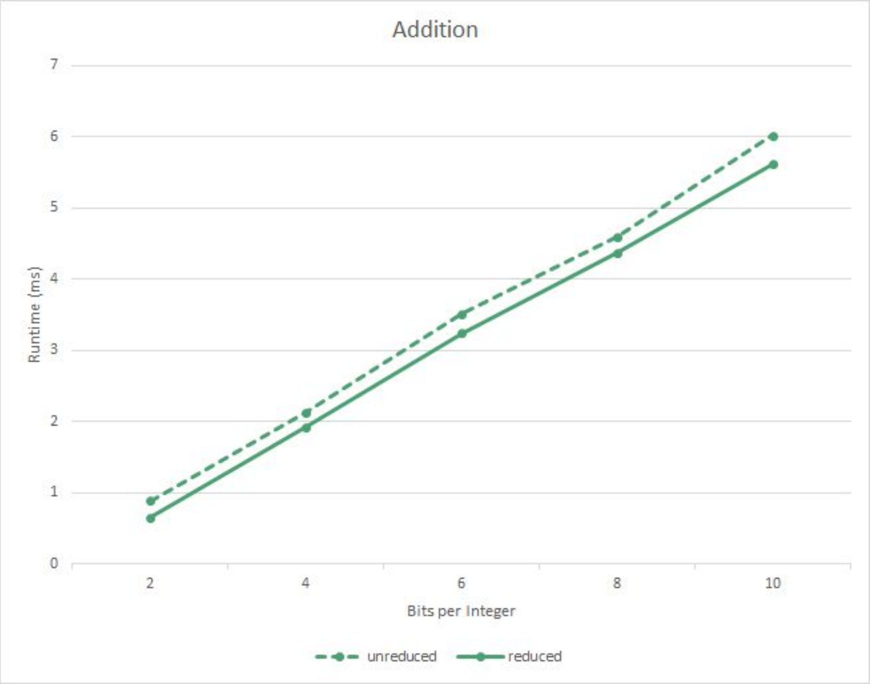
FHE Scheme Benchmarking



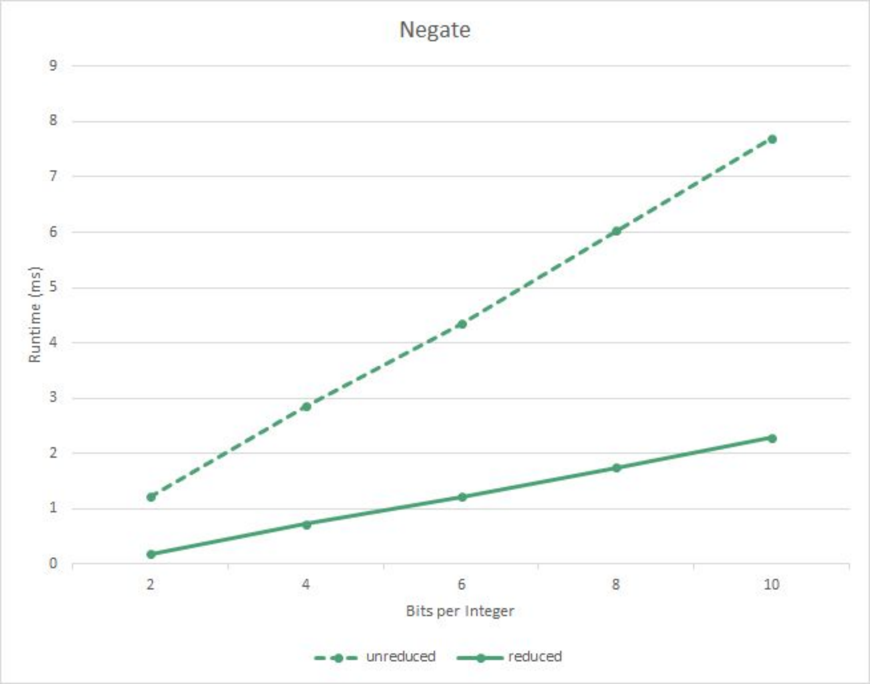


$O((n \lg q)^3)$

Optimization Benchmarking

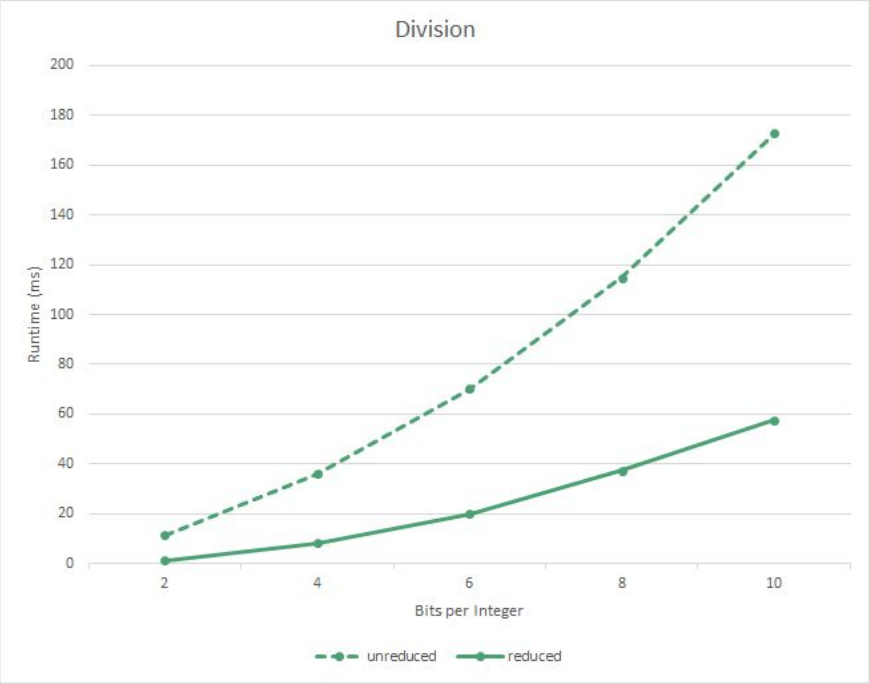


0.27 ms reduction

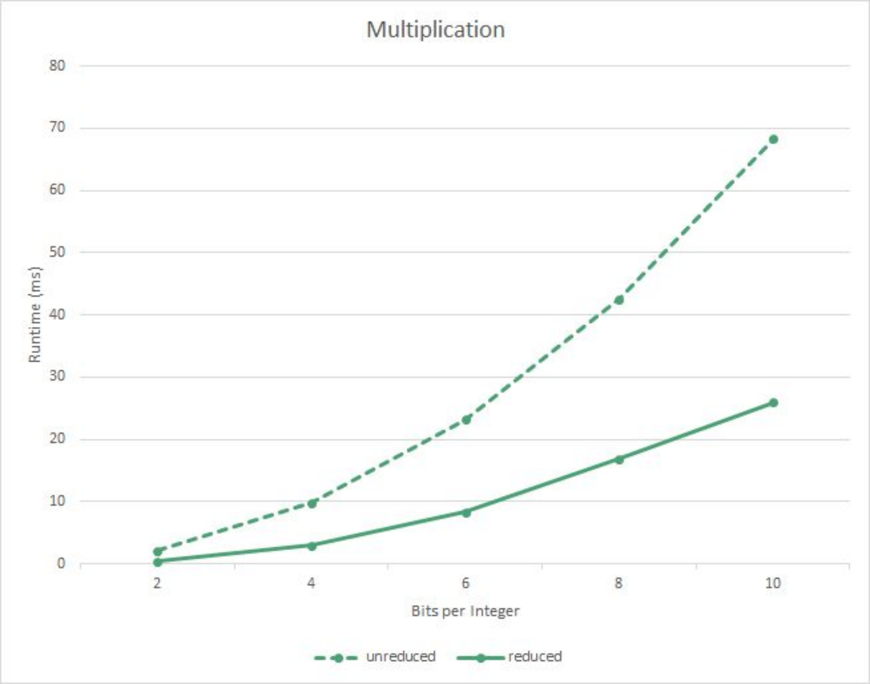


3.5 x reduction

Optimization Benchmarking

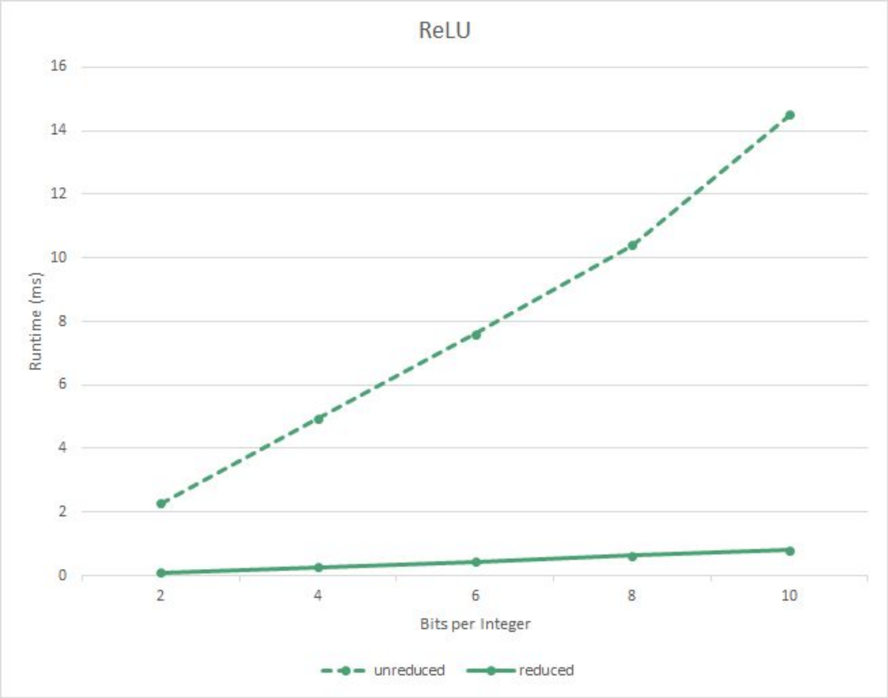


3.5 x reduction



2.8 x reduction

Optimization Benchmarking



17.5 x reduction

ReLU: (|x| + x) / 2

Conclusion

- A pipeline that speeds up the running of programs with fully homomorphic encryption
    - Internal representation that can be optimized with graph reductions
    - Scheduling and compiling homomorphic programs with Halide
- A basic API for easy use of the pipeline
- Demonstrated significant speedups compared to using bare fully homomorphic encryption

Future Work

- Adding heuristics to better handle larger function graphs
- Allowing function graphs to incorporate lower level FHE operations
- Adding new primitive gates (ex. MUX)
- Incorporate RLWE to allow faster arithmetic operations
- Improving the API

Acknowledgements

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