Mobile Sensor Networks:

Bounds on Capacity & Complexity of Realizability

Yizhen Chen[1]
Jesse Geneson[2]

[1]Princeton Int'l School of Math. & Science
[2]Dept. of Math., Iowa State University

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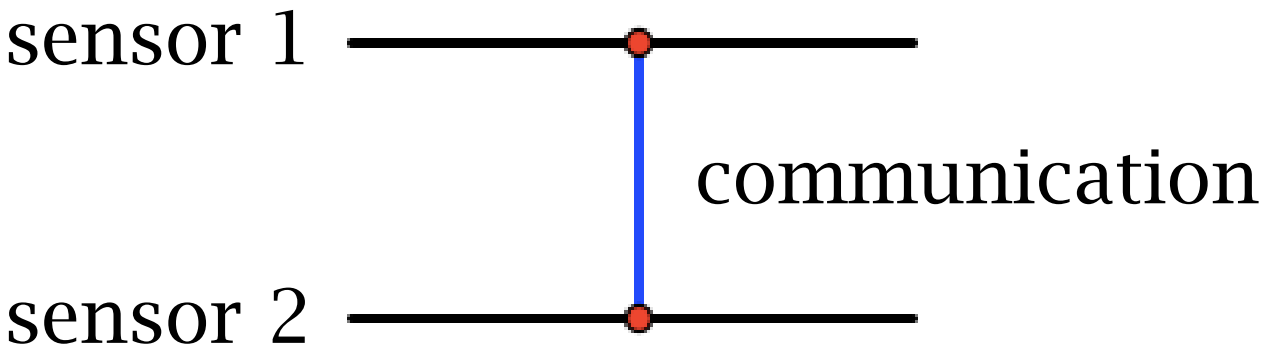
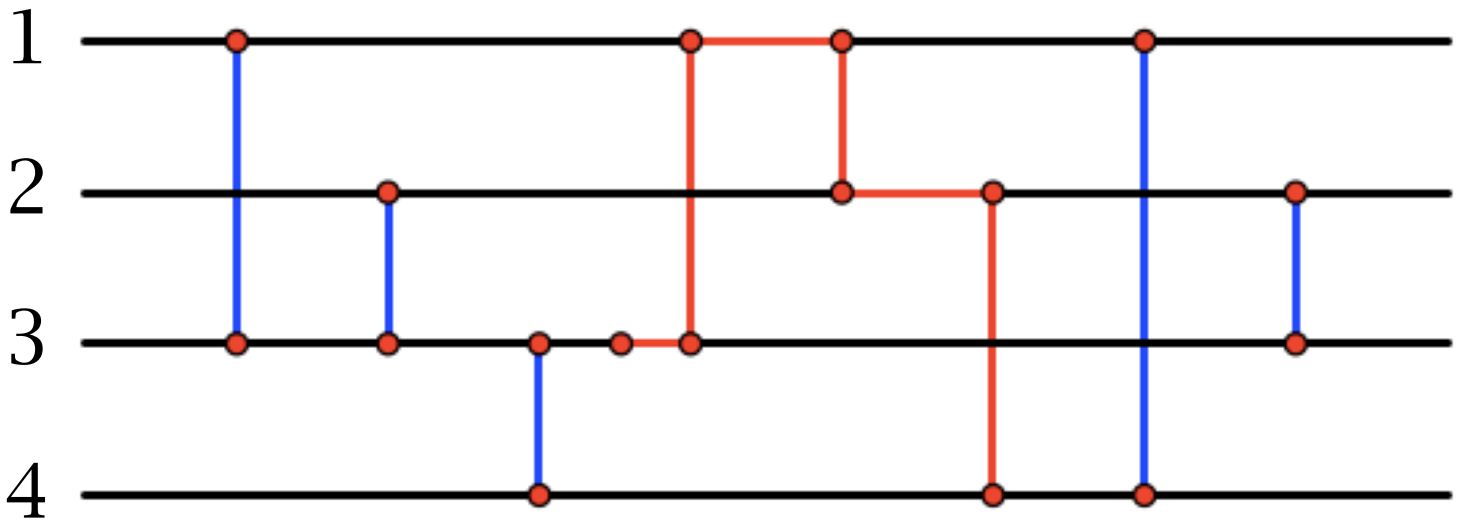
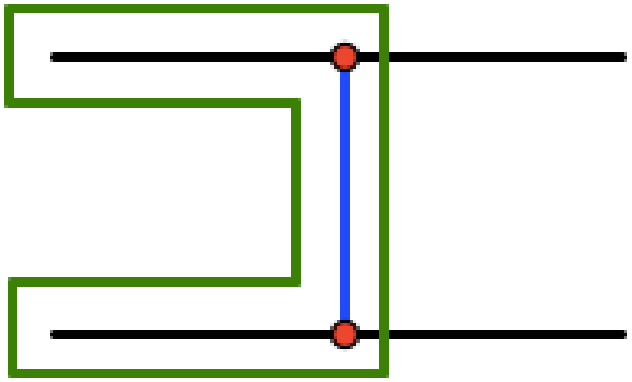
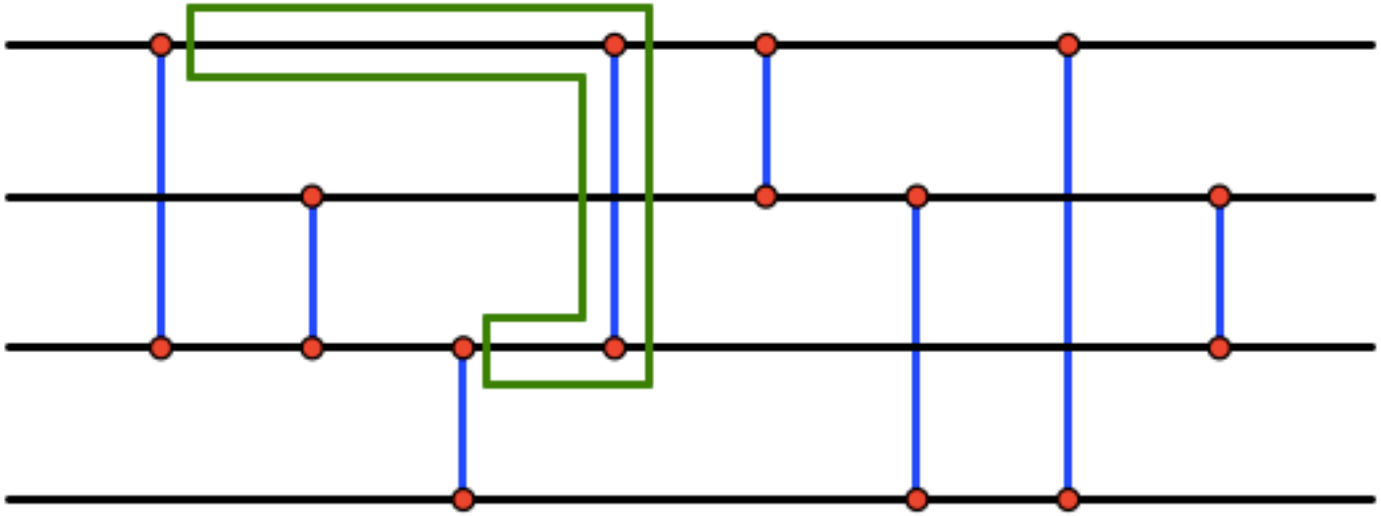
Mobile Sensor Networks (MSN)



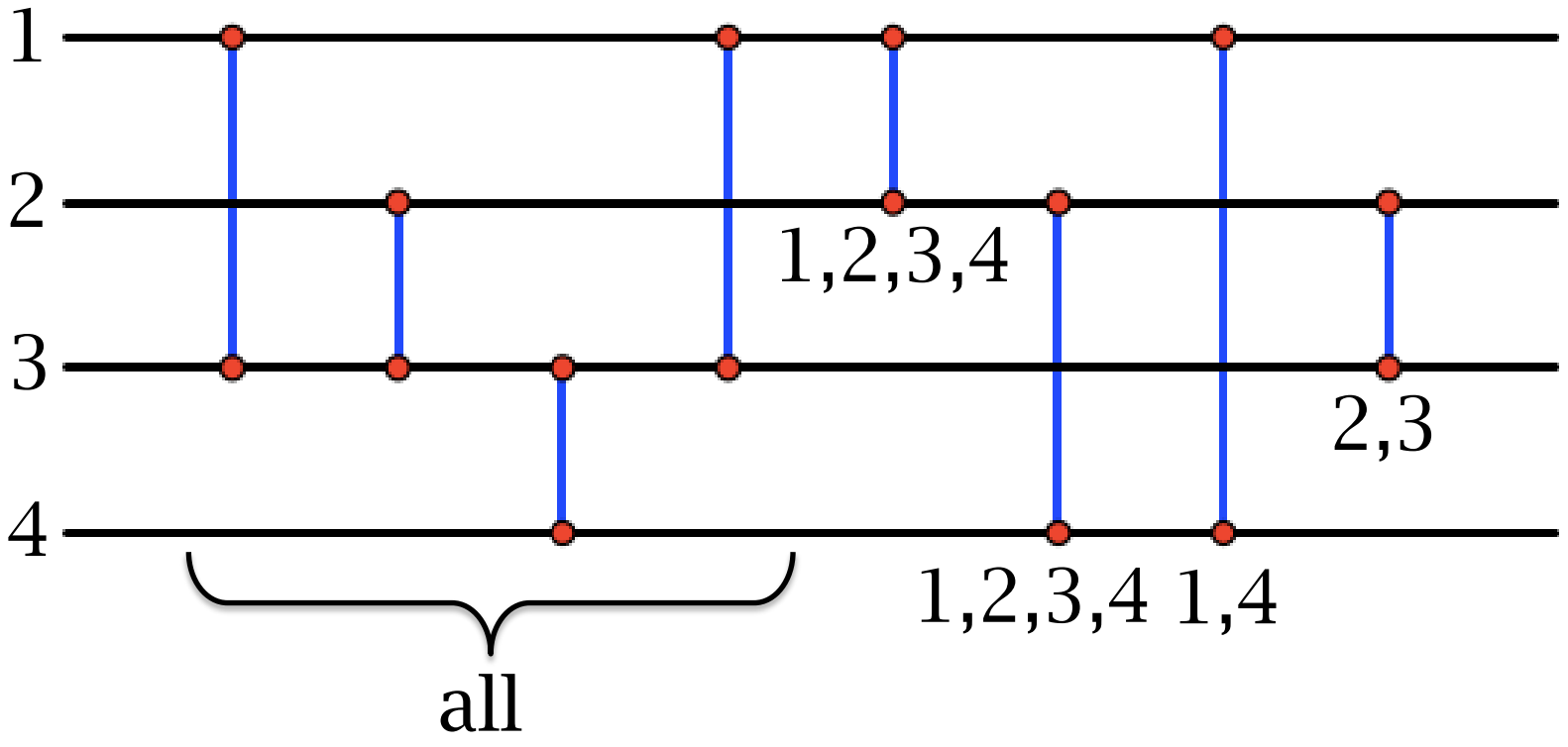


Information Packet



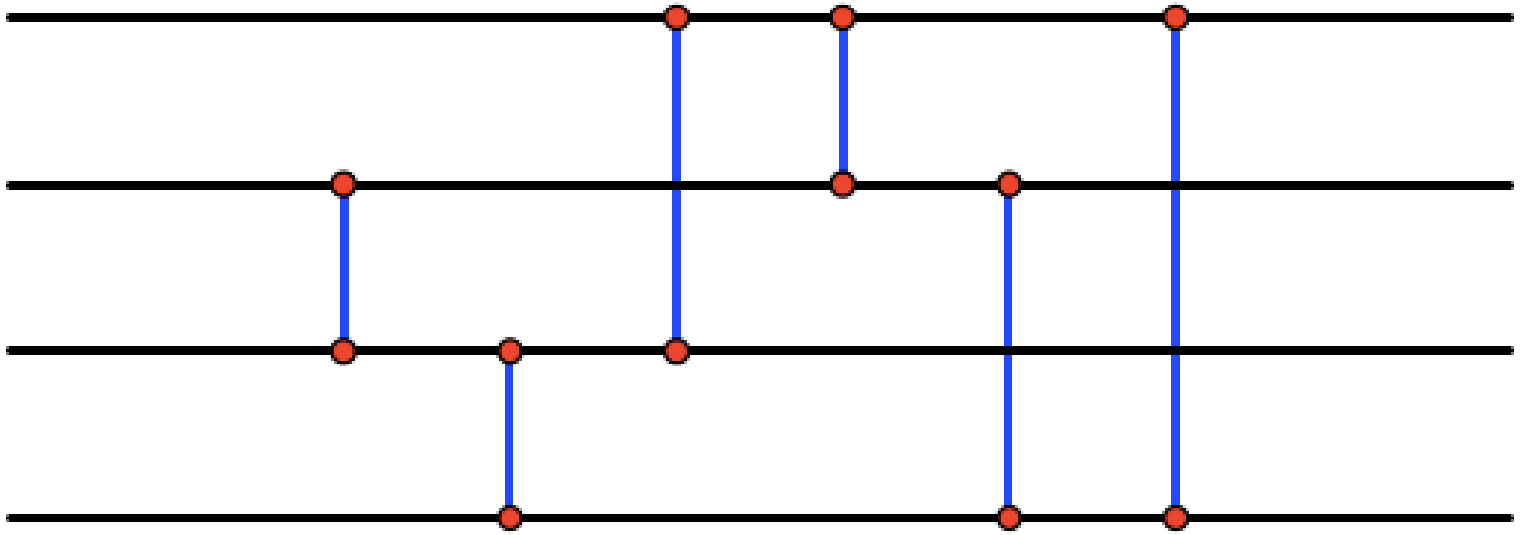


Capacity



$$\text{capacity} = \frac{\sum_{\text{packet } x} \#(\text{sensors that } x \text{ can reach})}{\#(\text{sensors}) \cdot \#(\text{packets})}$$

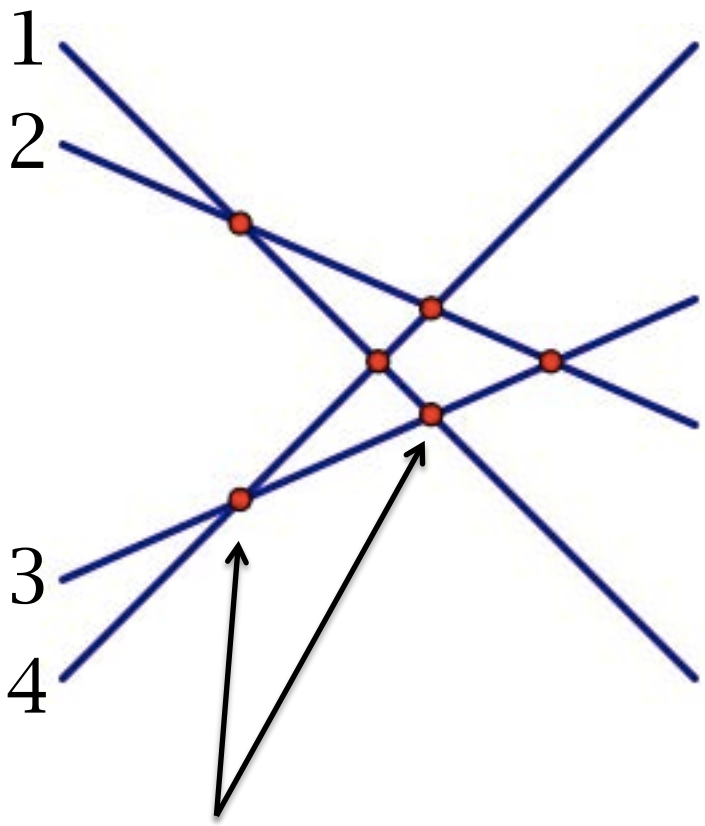
Restricted Combinatorial MSN



- Every pair communicates exactly once

- Expected Capacity?
- Minimum Capacity?
- Maximum Capacity?

Geometric MSN



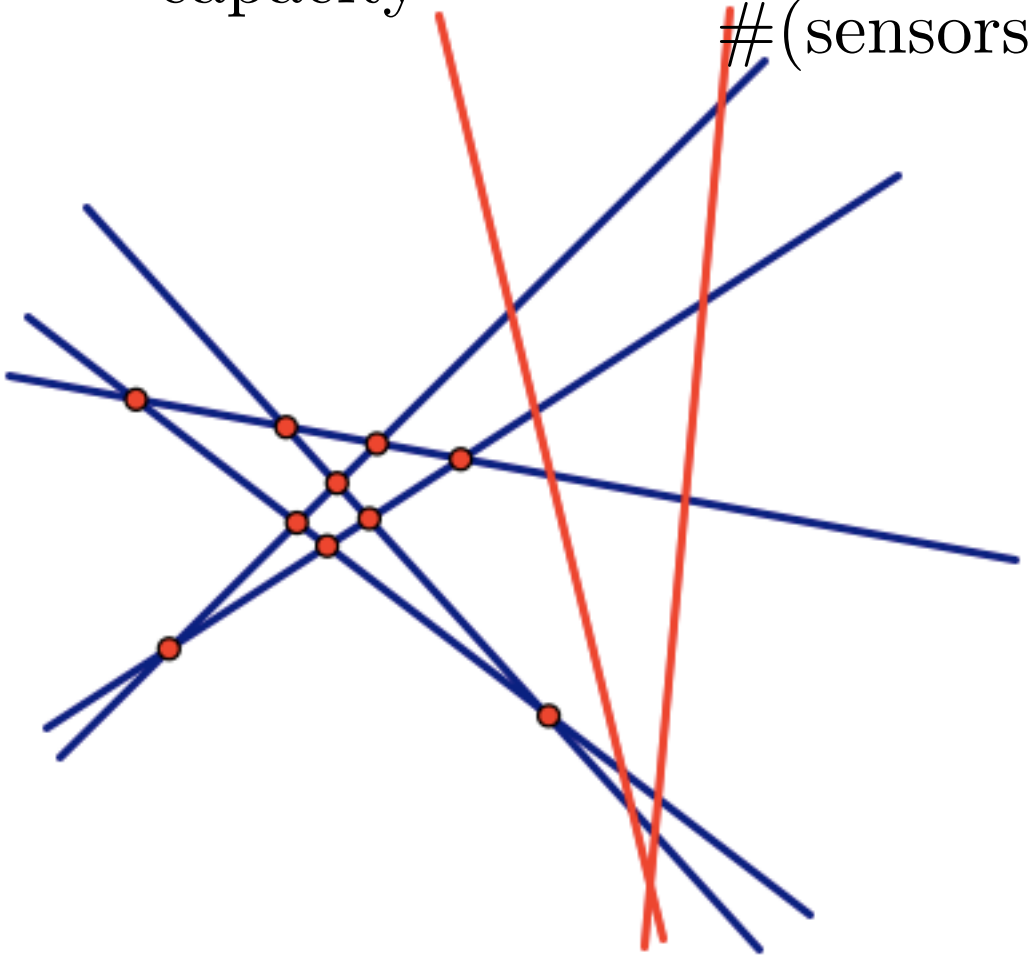
- Expected Capacity?
  - Gu et al.: ≤ 5/6
- Minimum Capacity?
- Maximum Capacity?

- Only *k* different slopes?

- Which RCMSNs are realizable as GMSNs?
  - *k* slopes?

RCMSN Maximum Capacity

$$\text{capacity} = \frac{\sum_{\text{packet } x} \#(\text{sensors that } x \text{ can reach})}{\#(\text{sensors}) \cdot \#(\text{packets})}$$



Last: = 2 sensors

2nd last: ≤ 3 sensors

3rd last: ≤ 4 sensors

. . .

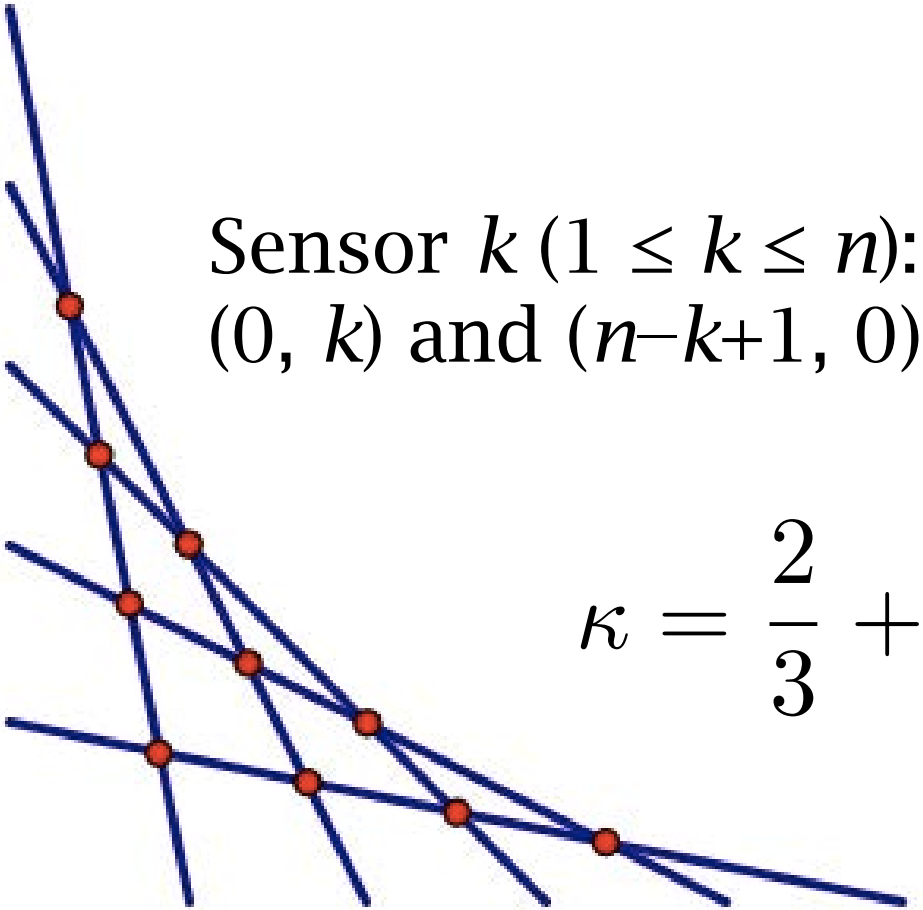
Example (GMSN):

- n–2 lines
- **collector**
- **distributor**

RCMSN Maximum Capacity

$$\kappa = 1 - \frac{1}{n} + \frac{2}{n^2}$$

RCMSN Minimum Capacity

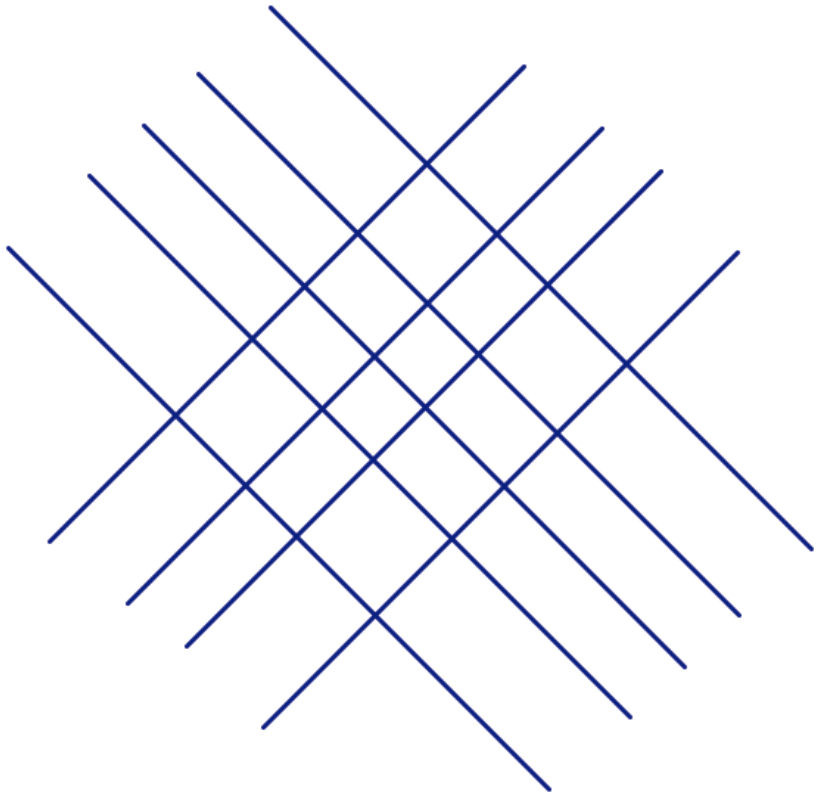


Sensor k ($1 \le k \le n$): connect $(0, k)$ and $(n-k+1, 0)$

$$\kappa = \frac{2}{3} + \frac{2}{3n}$$

k Slopes

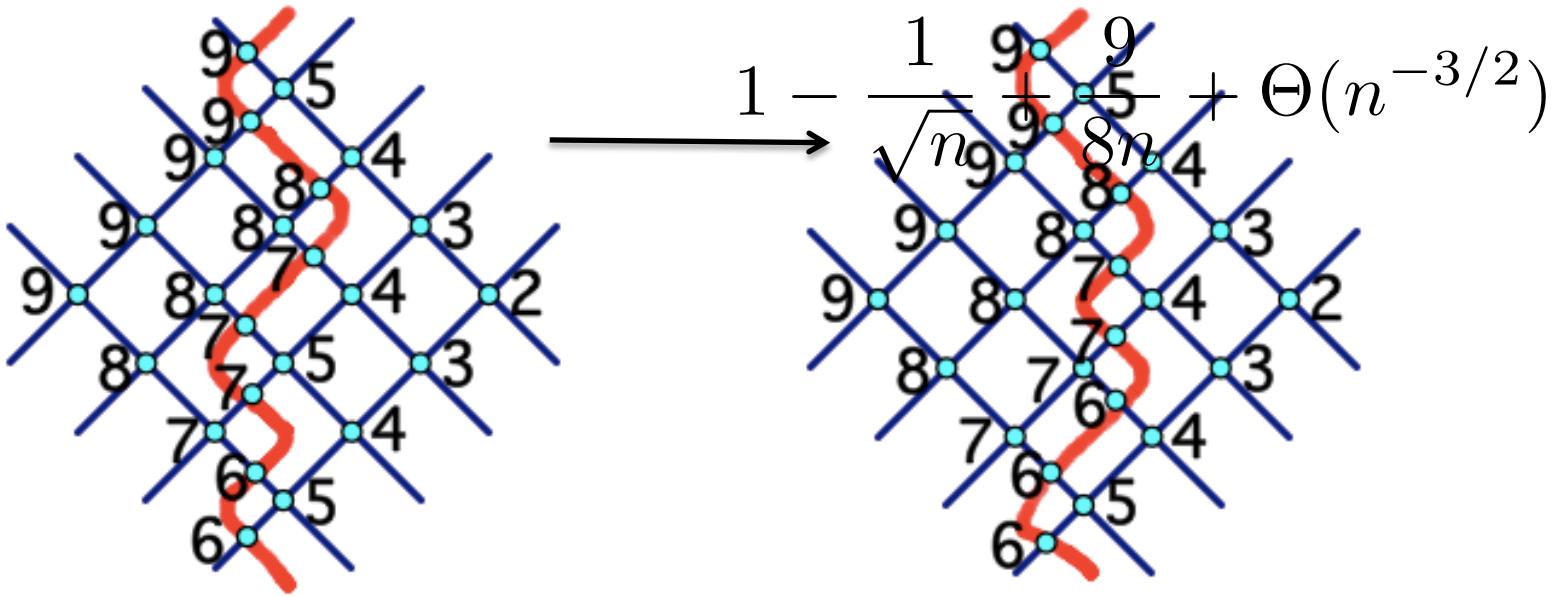
- Only 2 slopes
- An m-by-(n–m) grid
- Capacity:



$$\kappa = \frac{1}{nm(n-m)} \sum_{i=1}^{m} \sum_{j=1}^{n-m} (i+j) = \frac{1}{2} + \frac{1}{n}$$

k Slopes

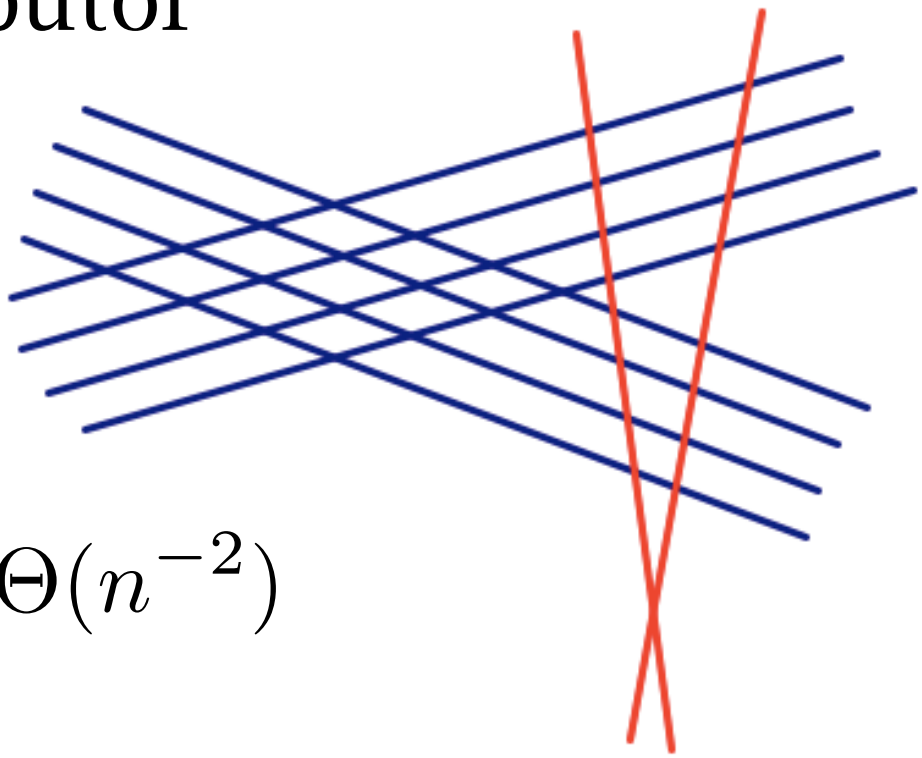
- Only 3 slopes
- Moving lines in grids
- M



k Slopes

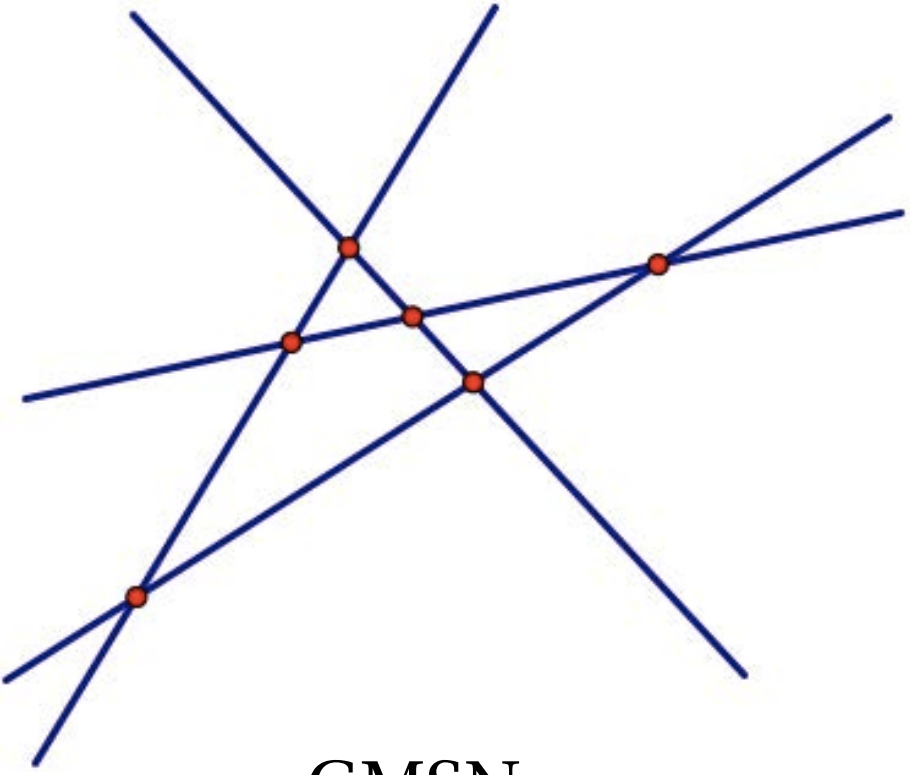
- Only $k \geq 4$ slopes

- Collector & distributor
  - k–2 slopes
  - collector
  - distributor

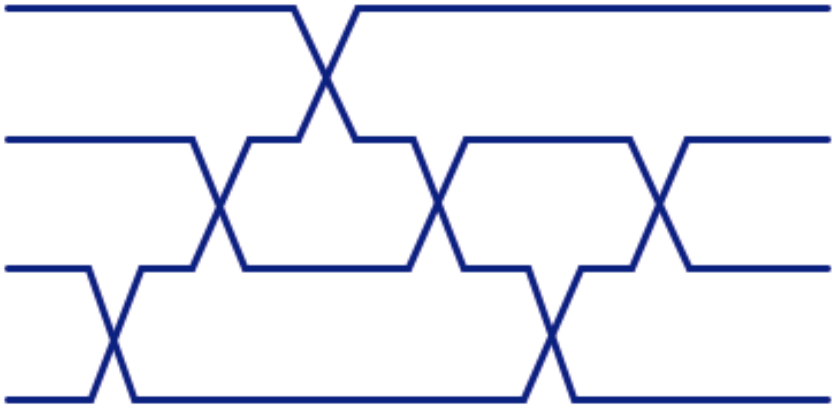


$$\kappa = 1 - \frac{k-2}{k-3}\frac{1}{n} + \Theta(n^{-2})$$

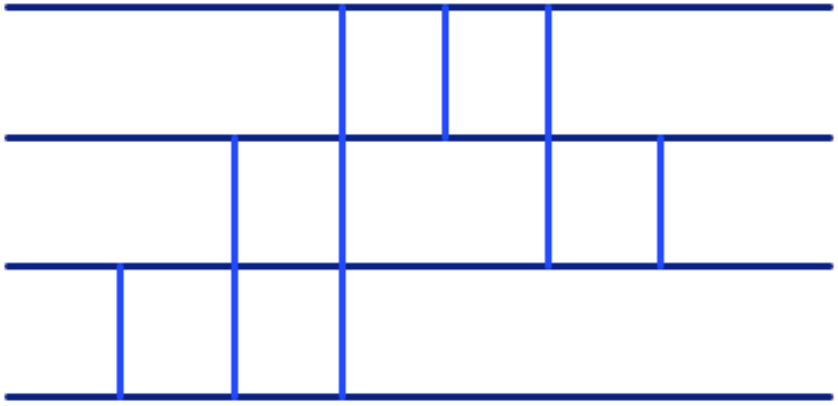
Realizability



GMSN

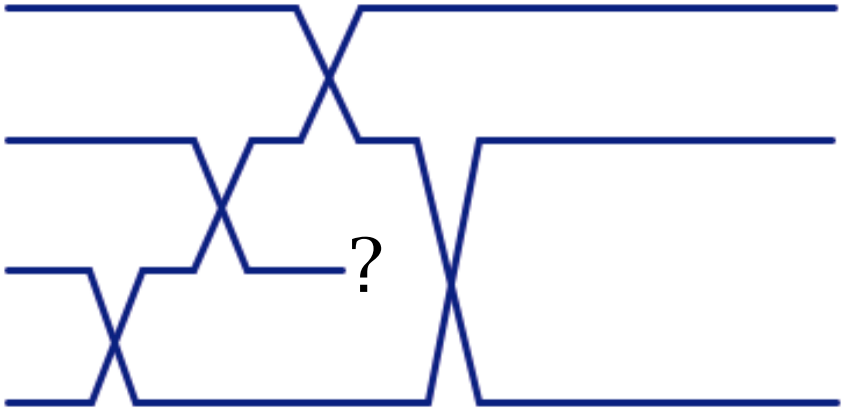


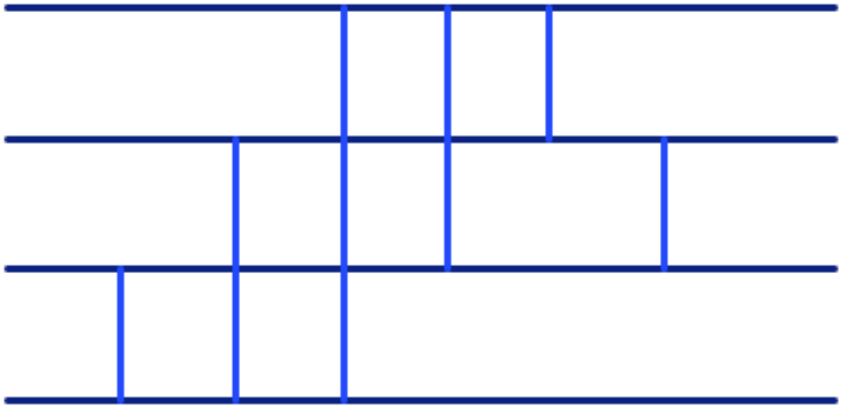
RCMSN



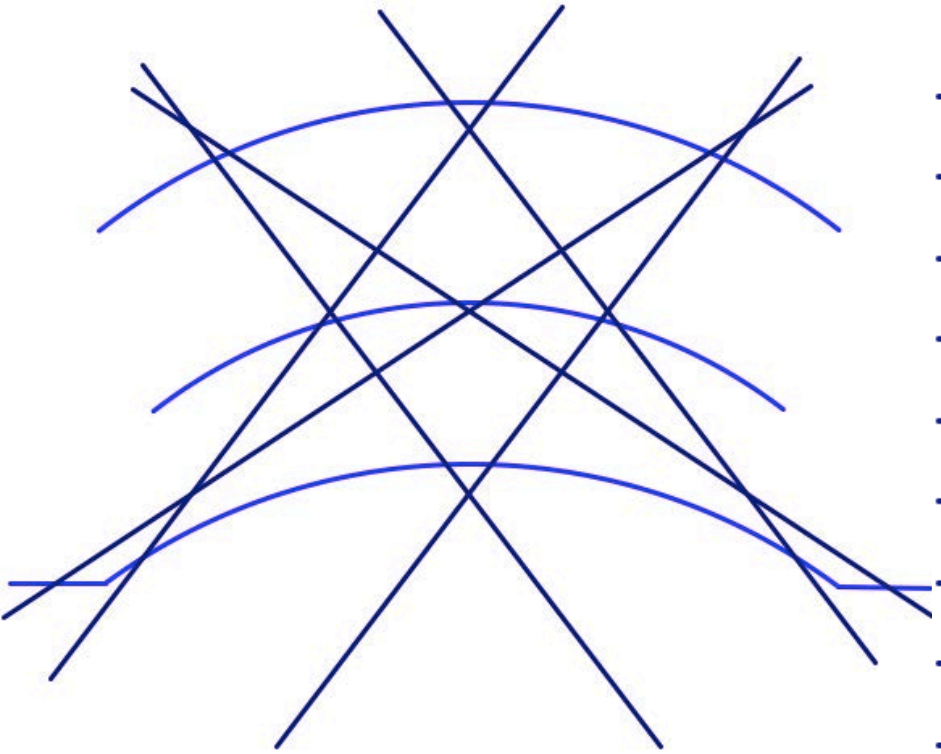
Realizability

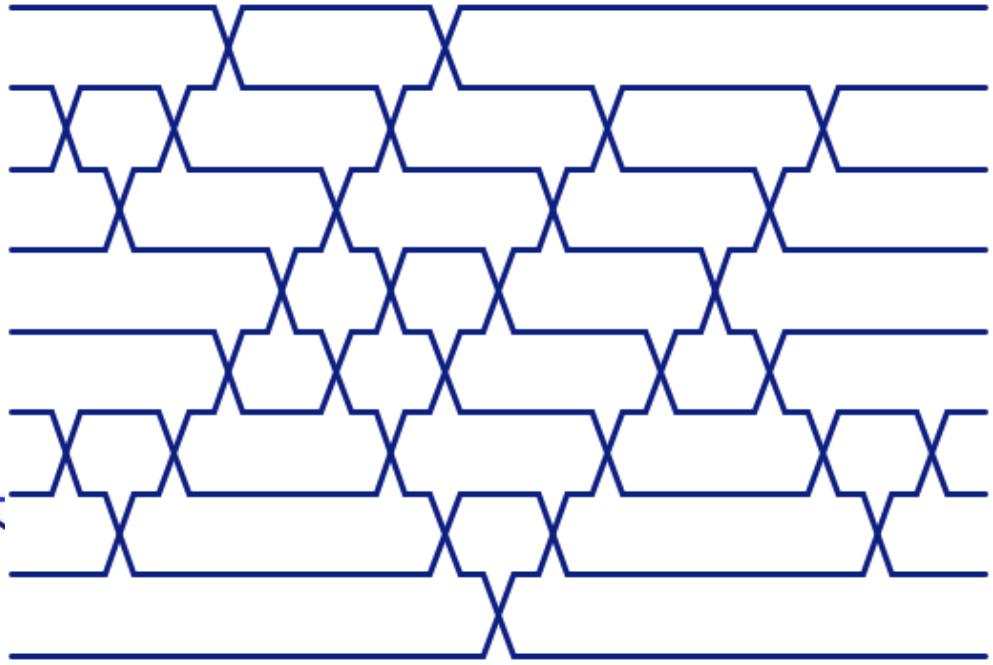
Not realizable
as GMSN





Realizability

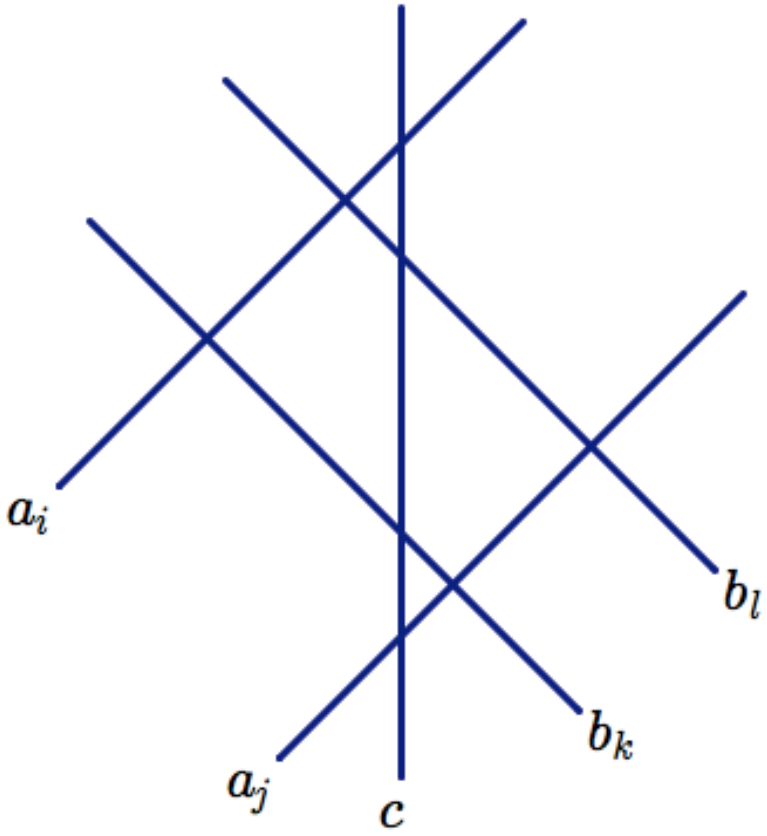




Contradicts Pappus Theorem
P. Shor (1991): Deciding stretchability is NP-Hard

Conclusion: Deciding if an RCMSN is realizable as a GMSN is NP-Hard

Realizability (3 slopes)



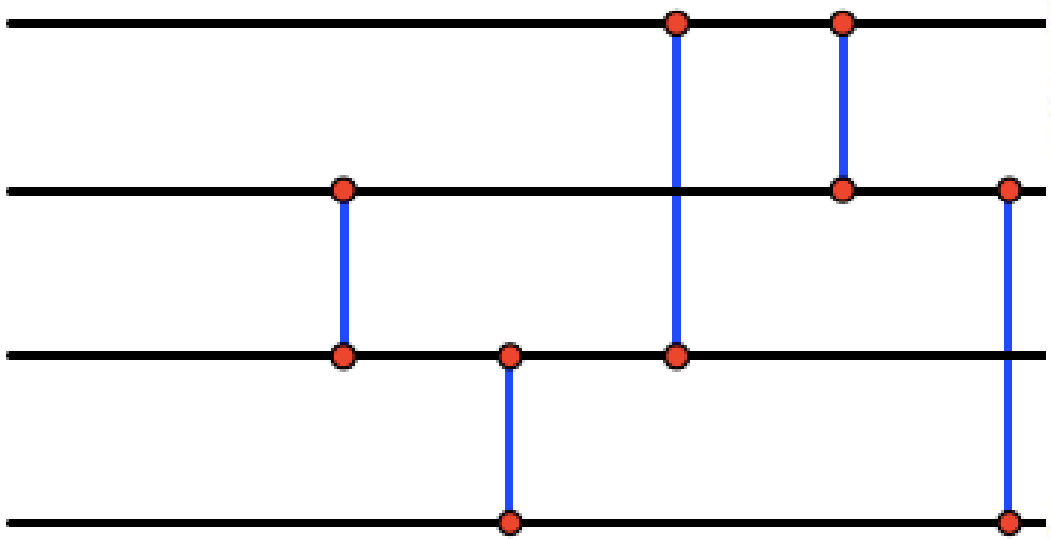
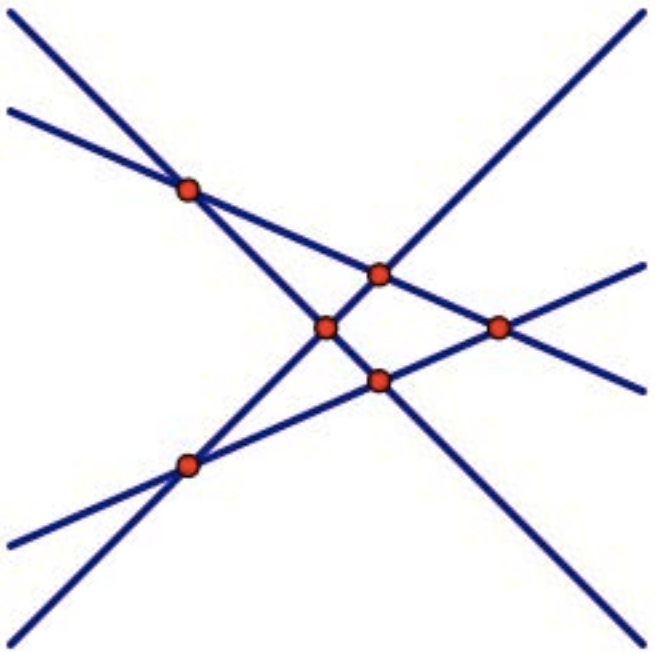
- Fix the slopes to ± 1 and ∞
- Linear programming

$$\operatorname{dist}(a_i, a_j) > \operatorname{dist}(b_k, b_l)$$

$$\operatorname{dist}(a_i, a_j) = \sum_{t=i}^{j-1} \operatorname{dist}(a_t, a_{t+1})$$

Conclusion: the realizability as GMSN in 3 slopes can be decided in polynomial time.

Future Work

- Realizability problem ($k \geq 4$ slopes)
- RCMSN Expected Capacity
- 


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Thank you for listening

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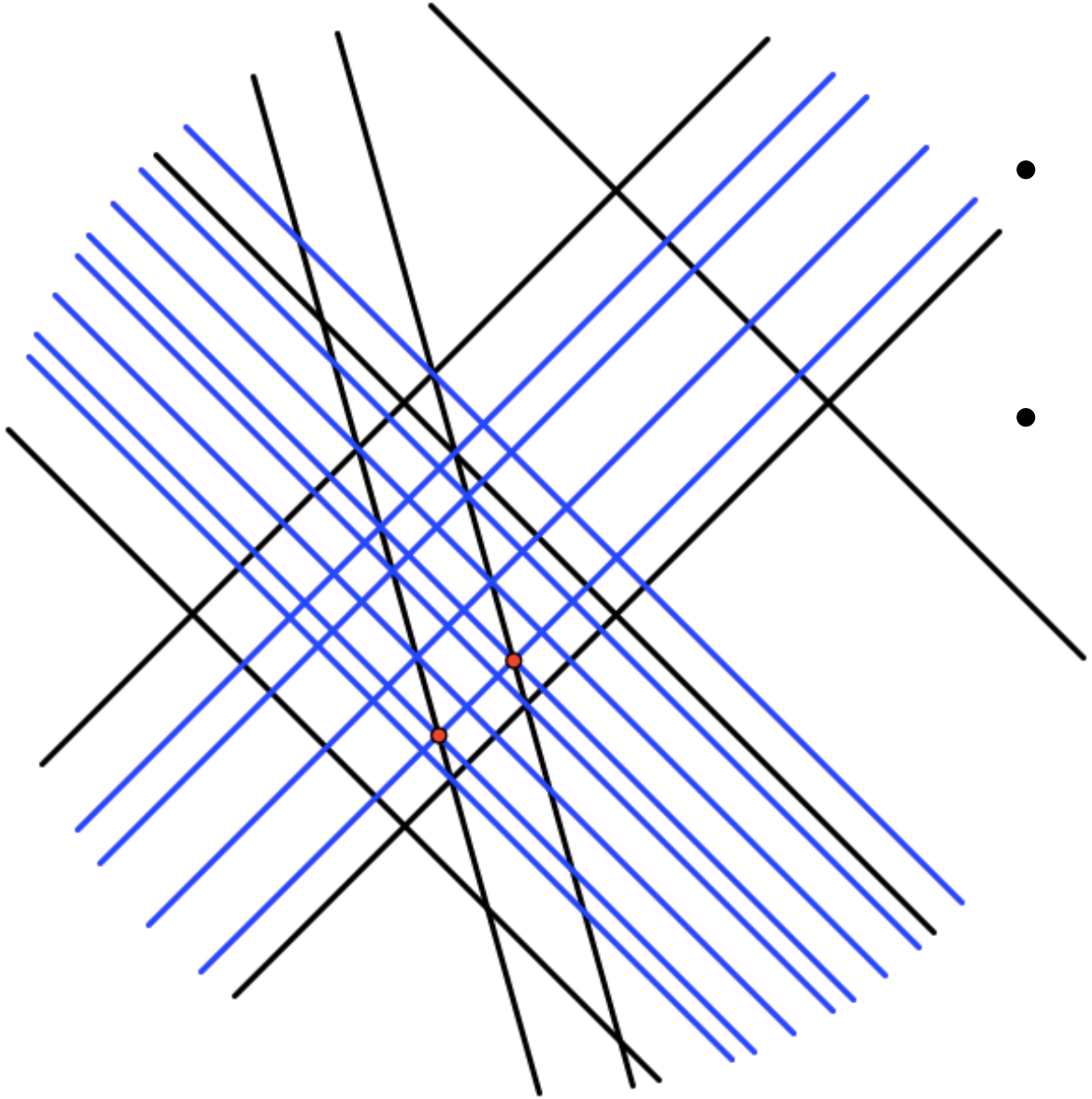
Realizability (3 slopes)



- How to fix the slopes?
- Add lines