

Introduction to Knot Theory

May 21

What is a knot?

A **knot** is a loop of string with no thickness, and is a closed curve in 3D space.

Introduction

Knot theory is the study of closed curves suspended in three dimensional space and how they can be deformed and categorized without passing through itself. One of the most basic knots is an unknot, which in its most basic form is simply a circle. The common definition of knots, like tying your shoelace isn't the same as knots in knot theory.

Unknot

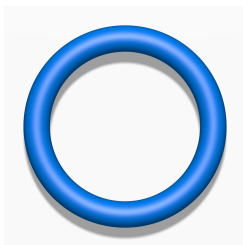


Figure: Unknot

Unknot

An unknot in its most simplest form looks like this.

Trefoil



Figure: Trefoil

Another common knot is the trefoil which is the second most simplest knot while in it's basic form.

Projections

Definition

A **projection** is an image of a knot in 2D space.

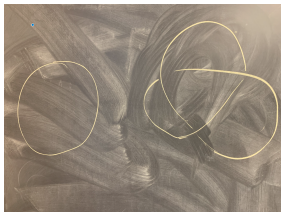


Figure: These are projections of the unknot (left) and trefoil (right).

Projections

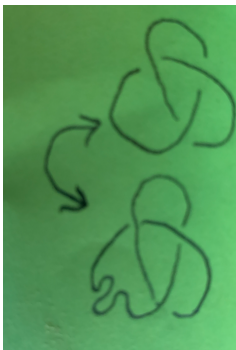
Details

There are infinite projections of each knot, because knots are actually tangled loops in 3D. When we show them on paper they are just a picture of a knot from a certain angle, while the knot is tangled in a certain way. The ways that we can switch between projections in 2D are called planar isotopy and Reidemeister moves.

Planar Isotopy

Definition

Planar Isotopy is the deformation of a knot within the projection plane.



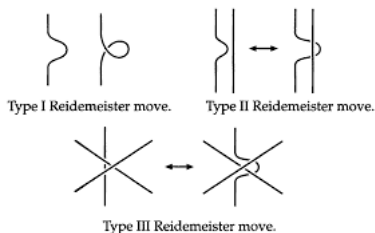
Reidemeister moves

There are three other ways to deform a knot and these moves are called Reidemeister moves.

Definition

Reidemeister moves are the three ways to change the crossings of a knot, while keeping the knot the same.

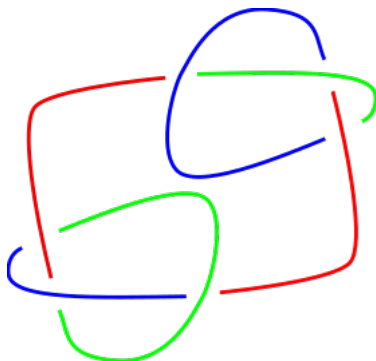
These moves look like this:



Tricolorability

Definition

The tricolorability of a knot is the ability of a knot to be colored with three colors subject to certain rules.



Tricolorability and Reidemeister Moves

All reidemeister moves preserve tricolorability.

Example

Reidemeister moves using tricolorability.

Because reidemeister moves and planar isotopy preserve tricolorability on a projection we can prove that the trefoil and unknot are two different knots because the projections of the unknot are not tricolorable while the projections of the trefoil are.

Orientation

Definition

Orientation is the direction that we choose to travel around a knot. To show this we place arrows on the knot.

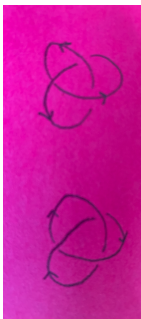


Figure: These are the two ways to orient a trefoil knot. ◀ ≡ ▶ ≡ ↺ ↻

Did you know?

You may be wondering if we can apply knot theory to anything in our lives, well we can. At first people thought that knot theory explained the structure of atoms, but that hypothesis was proved to be incorrect. One application for knot theory however is right inside you, your DNA! DNA or deoxyribonucleic acid is formed in a structure called a **double helix**.

Double helix

Definition

Double helix's are a pair of helix's intertwined around a line.

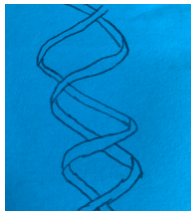


Figure: A double helix.

Topoisomerases

Topoisomerases

Enzymes called **Topoisomerases** can change the topology of DNA. If the DNA that the topoisomerases alter is in a loop then these enzymes would also change the type of knot or link the DNA is configured in. There are three main ways these enzymes are able to change the topology of DNA

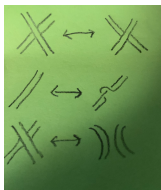


Figure: Most common actions for enzymes to take on DNA.