Fast GPU Accelerated Ising Models for Practical Combinatorial Optimization

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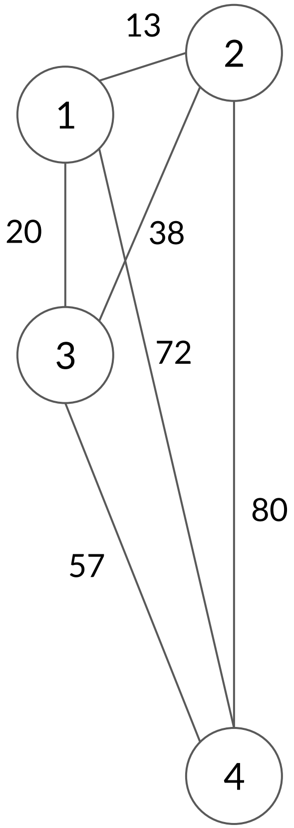
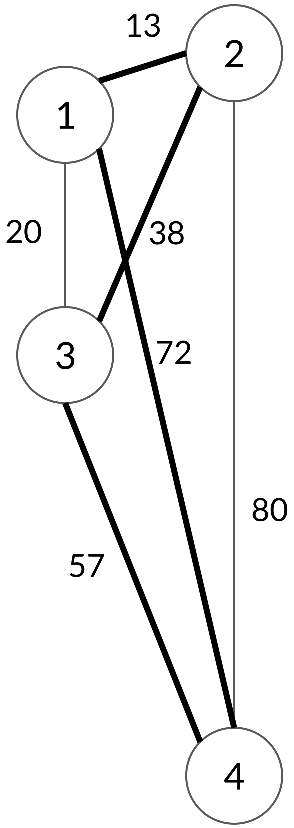
Combinatorial Optimization



Finding the best solution from a finite set of possible solutions

Example:
Traveling Salesman Problem

Combinatorial Optimization



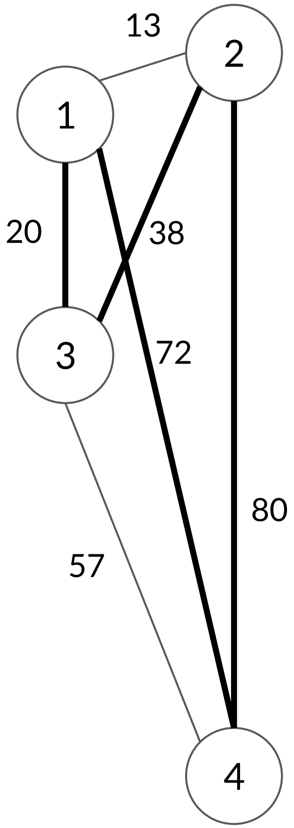
Finding the best solution from a finite set of possible solutions

Example:
Traveling Salesman Problem

1 - 4 - 3 - 2 - 1: 180

Combinatorial Optimization

Finding the best solution from a finite set of possible solutions

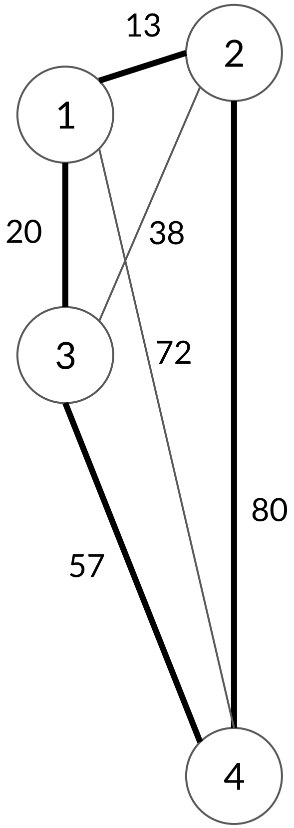


Example:
Traveling Salesman Problem

1 - 4 - 3 - 2 - 1: 180

1 - 3 - 2 - 4 - 1: 210

Combinatorial Optimization



Finding the best solution from a finite set of possible solutions

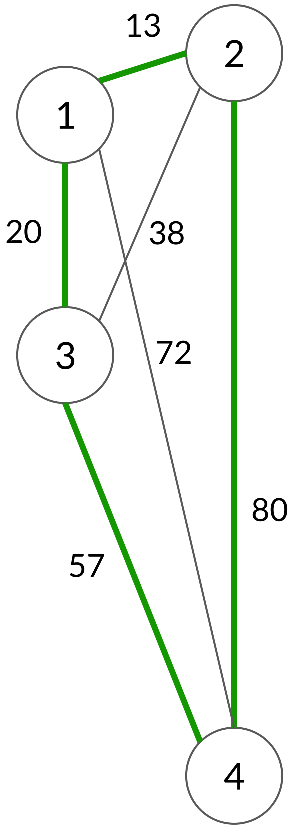
Example:
Traveling Salesman Problem

1 - 4 - 3 - 2 - 1: 180

1 - 3 - 2 - 4 - 1: 210

1 - 2 - 4 - 3 - 1: 170

Combinatorial Optimization



Finding the best solution from a finite set of possible solutions

Example:
Traveling Salesman Problem

1 - 4 - 3 - 2 - 1: 180

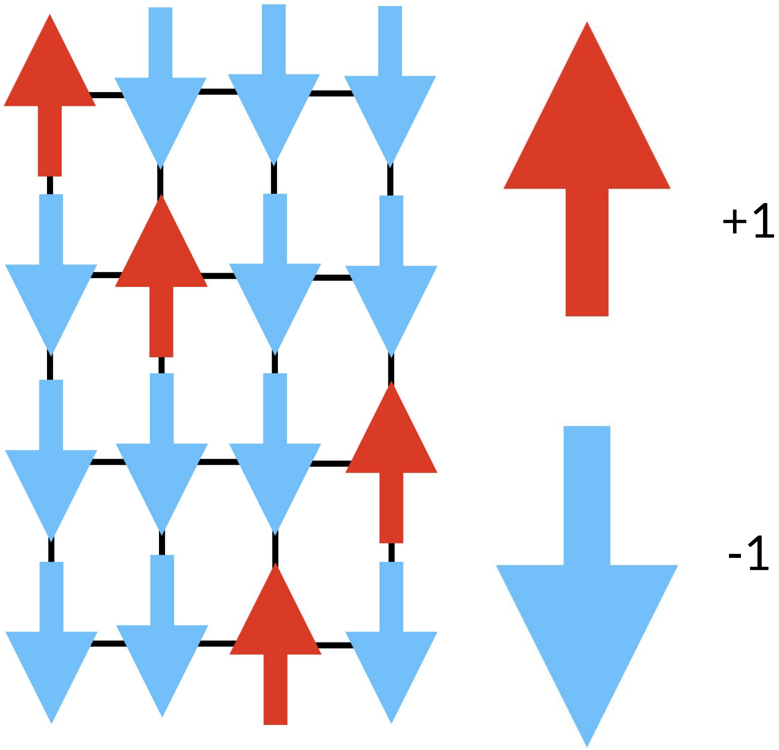
1 - 3 - 2 - 4 - 1: 210

1 - 2 - 4 - 3 - 1: 170

- NP-Hard → no known fast exact algorithms, but still want to solve
- Many applications, e.g. biotech & finance
- Solver needs to be flexible enough for many problems, but also structured enough to be efficient

Ising Model

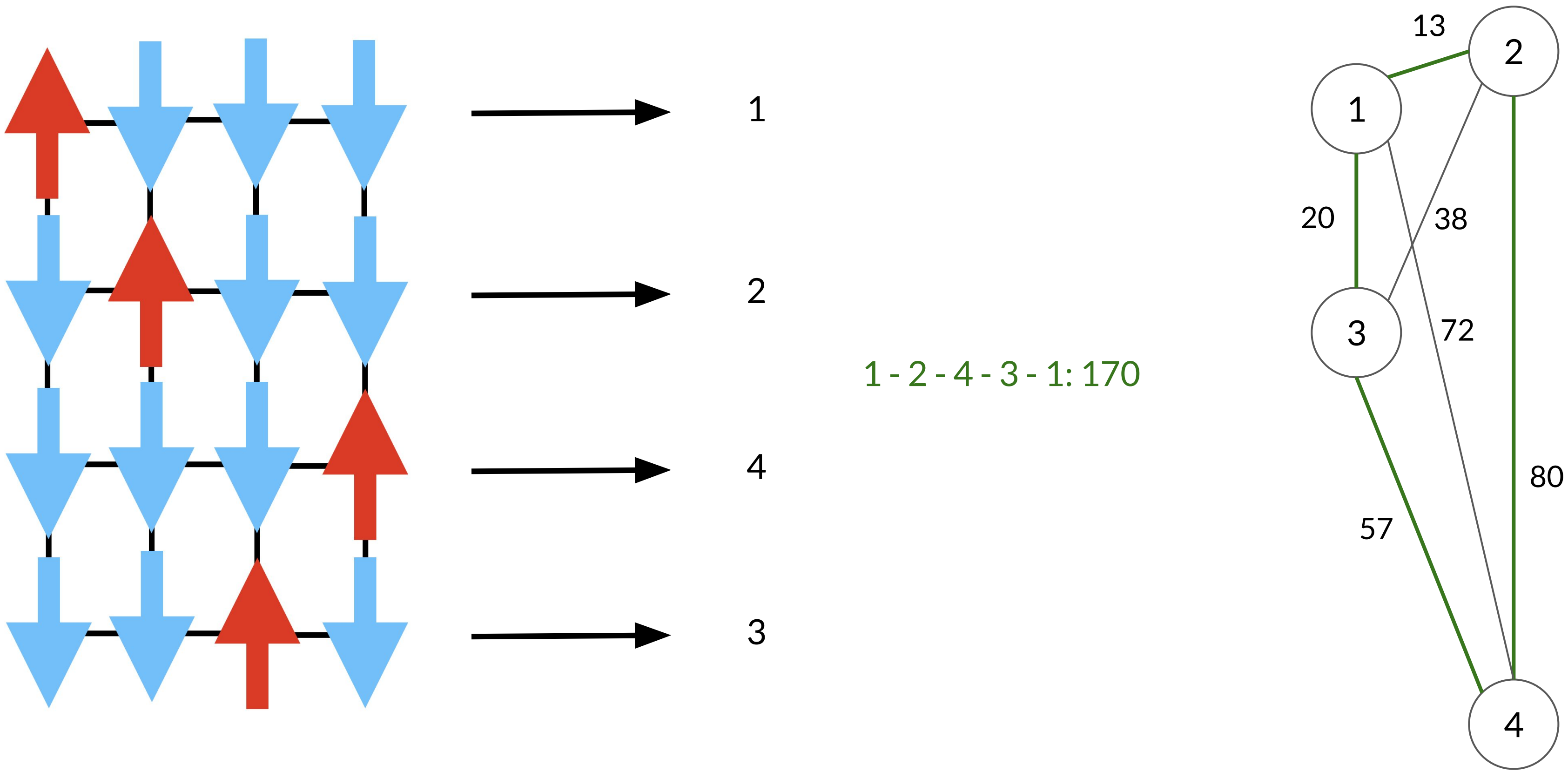
A physical representation of interactions between magnetic particles



Ising Model

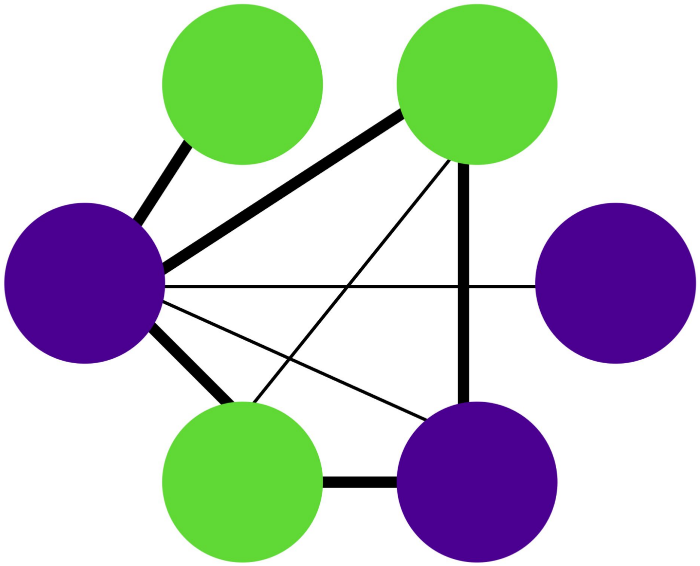
A physical model of interaction between magnetic particles

A robust and flexible representation of combinatorial optimization



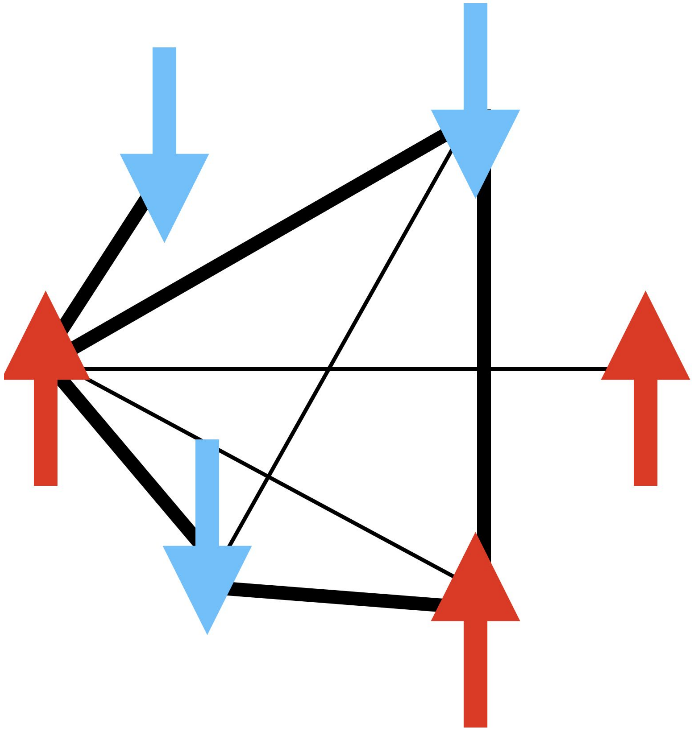
MAXCUT

A problem that maps directly to the Ising Model



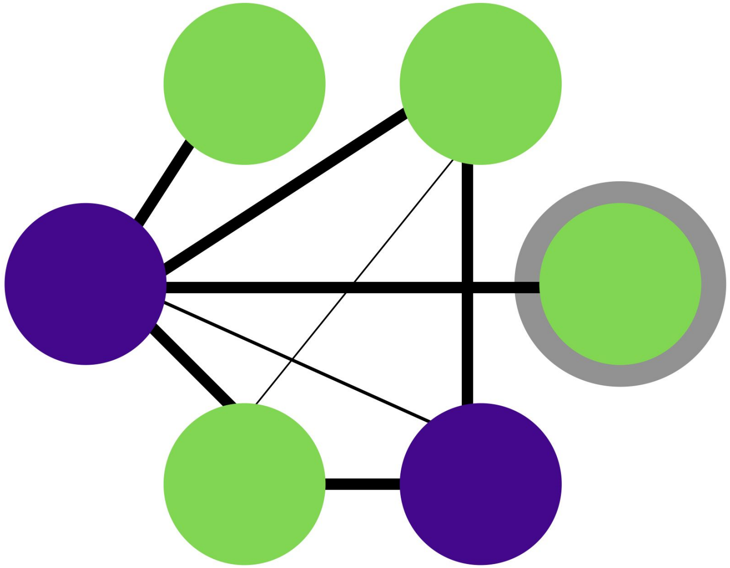
Cut: 5

=



MAXCUT

A problem that maps directly to the Ising Model



Cut: 6

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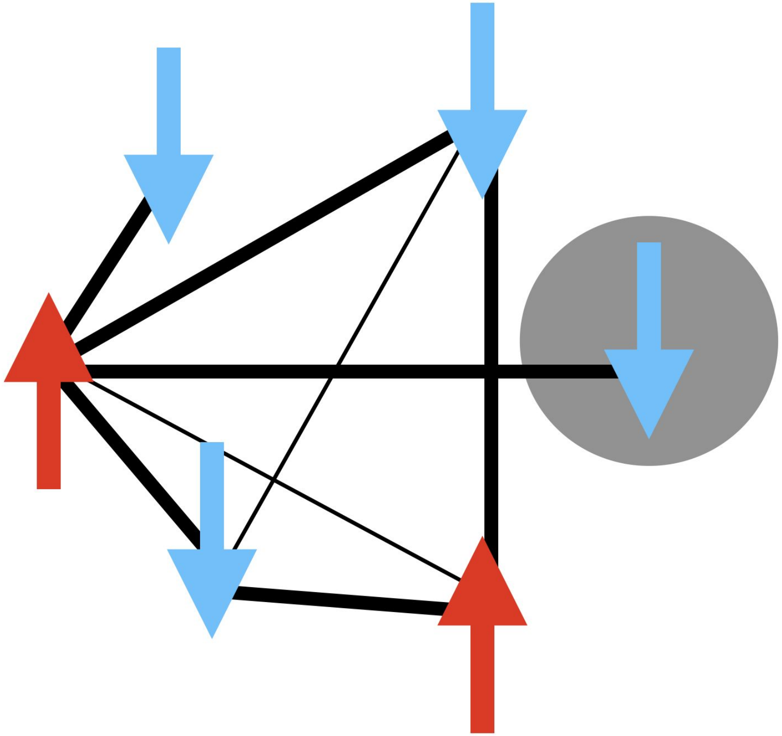


Prior Work

Many attempts, but none are optimal

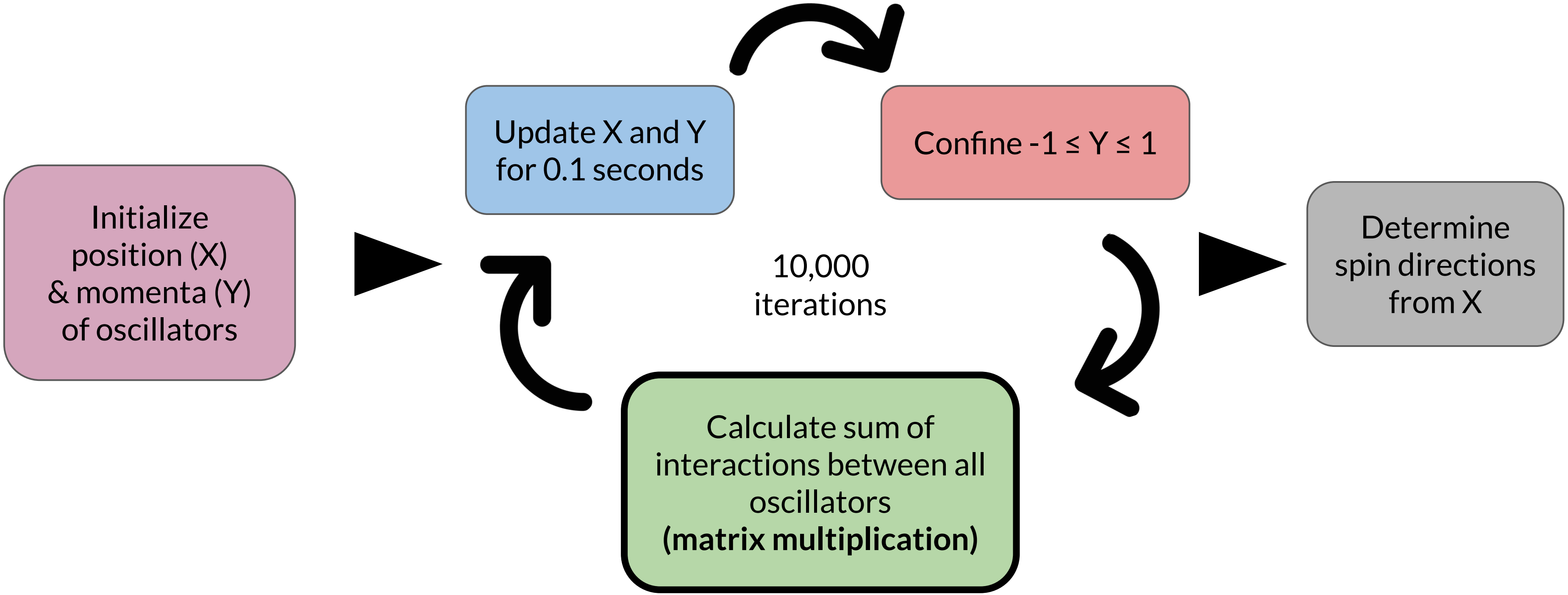
1. Simulated Annealing (SA): Flips spins one at a time until the cut is maximized. → Sequential, long runtime for poor solution quality

2. Parallel Tempering: Runs several (~8) SA instances in parallel. → Marginally superior to SA, but suffers from same problems

3. Simulated Bifurcation: Simulates a network of nonlinear optical oscillators → Higher solution quality, but current implementations are not optimized for real life problems

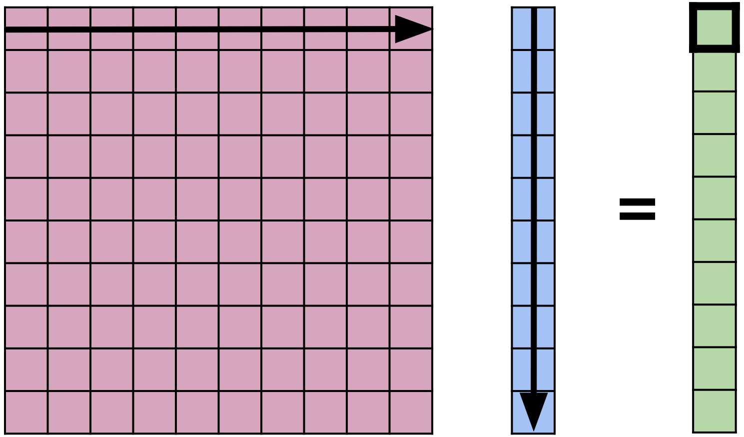
Simulated Bifurcation

Classical simulation of a quantum phenomena

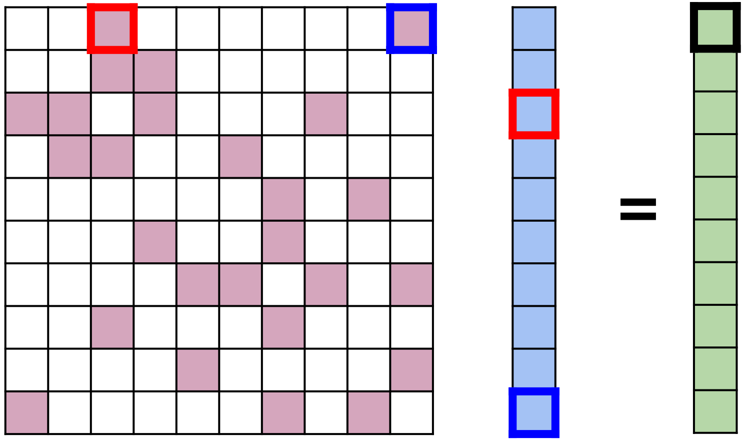


Sparsity in Matrix Multiplication

Real life problems are sparse — we take advantage of this to get speedups



10x10 Dense Matrix: **10** multiplications per value



10x10 Sparse Matrix: **2** multiplications per value

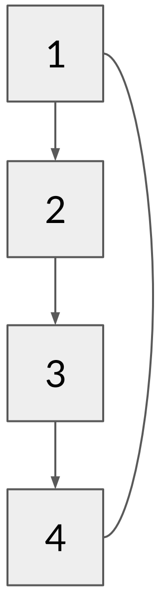
GPU Computing

Extremely parallel solving

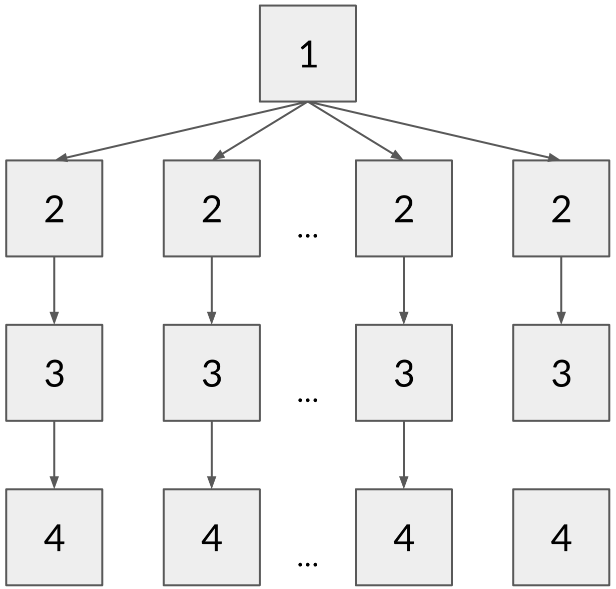
GPU Computing

Extremely parallel solving

Sequential Program Design
(CPU: easy, but slow)



Parallel Program Design
(GPU: hard, but fast)



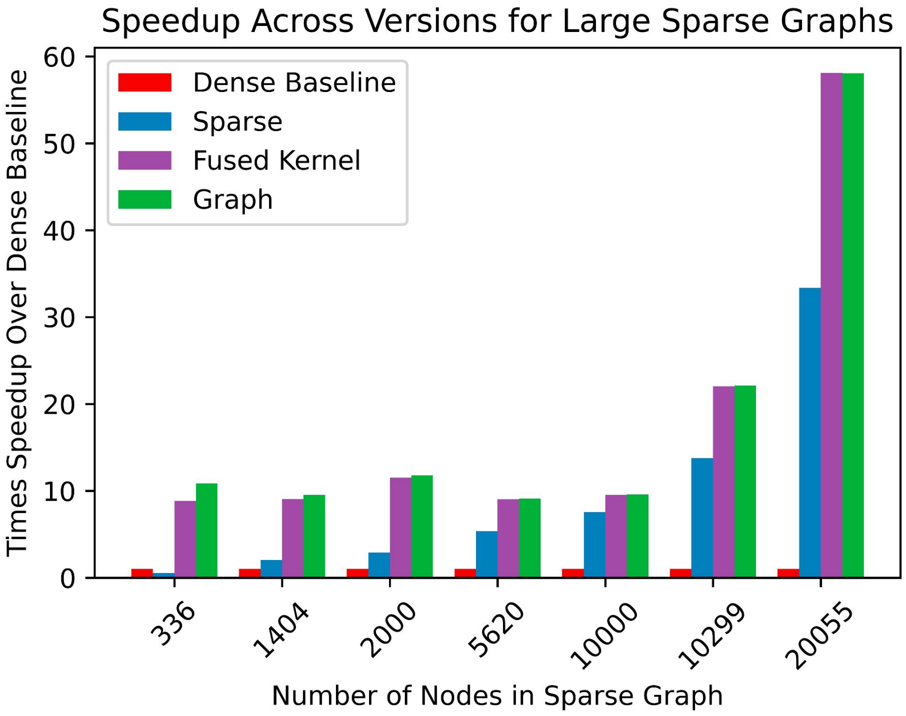
Four Development Versions

Optimization steps to final algorithm

Dense Baseline	No optimizations
Sparse	Matrix multiplication redesigned to take advantage of sparsity
Fused Kernel	Fuses 3 separate steps (update, confine, interactions) into one GPU kernel
Graph	Chains all kernel calls together using CUDA Graph

Results: MAXCUT Speedup (Real Life)

Relative time to 10,000 steps for a representative sample of graphs

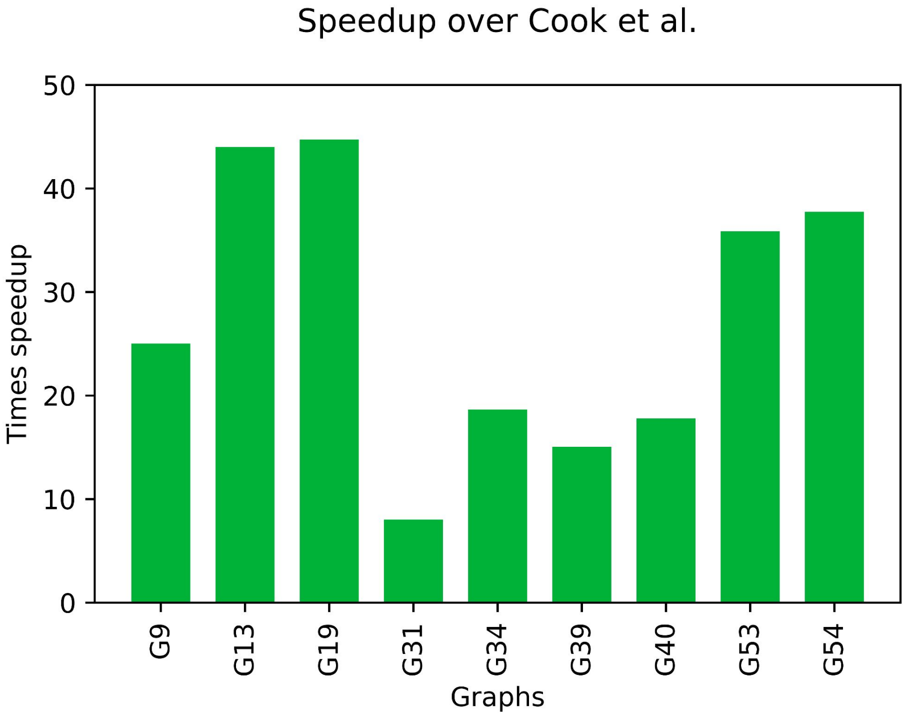


Gmean speedup: 14.5x faster
Max speedup: 58.1x faster

Ageron, R., Bouquet, T., & Pugliese, L. (2023). *Simulated Bifurcation (SB) algorithm for Python* (1.2.0).

Results: MAXCUT Simulated Annealing Speedup

Comparison of Time-To-Solution for a representative sample of graphs

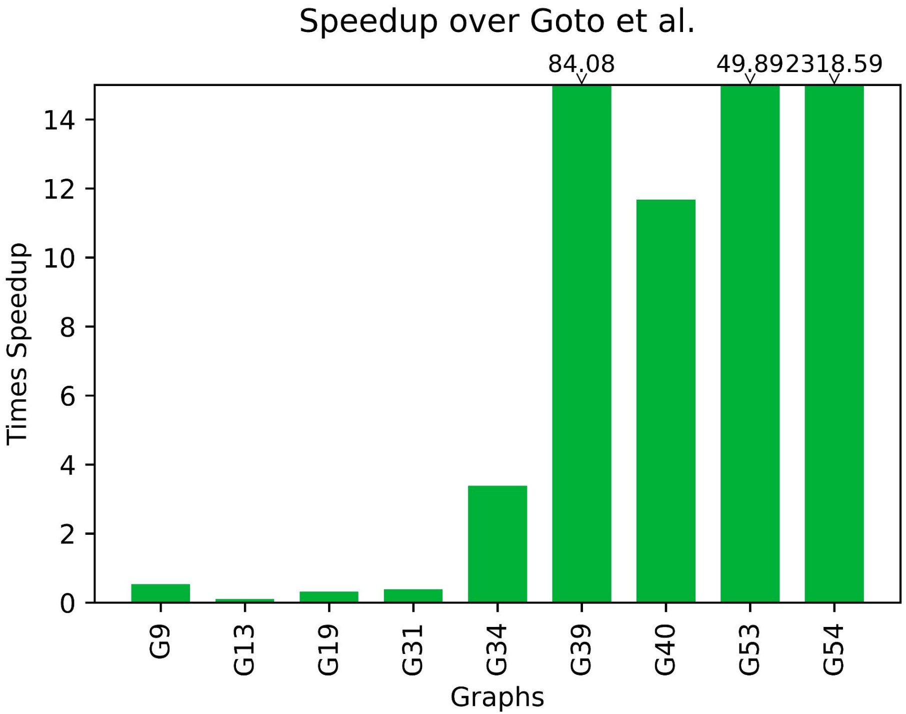


Gmean speedup: 25.5x faster
Max speedup: 44.7x faster

Cook, C., Zhao, H., Sato, T., Hiromoto, M., & Tan, S. X.-D. (2019). *GPU Based Parallel Ising Computing for Combinatorial Optimization Problems in VLSI Physical Design*.

Results: MAXCUT Simulated Bifurcation Speedup

Comparison of Time-To-Solution for a representative sample of graphs



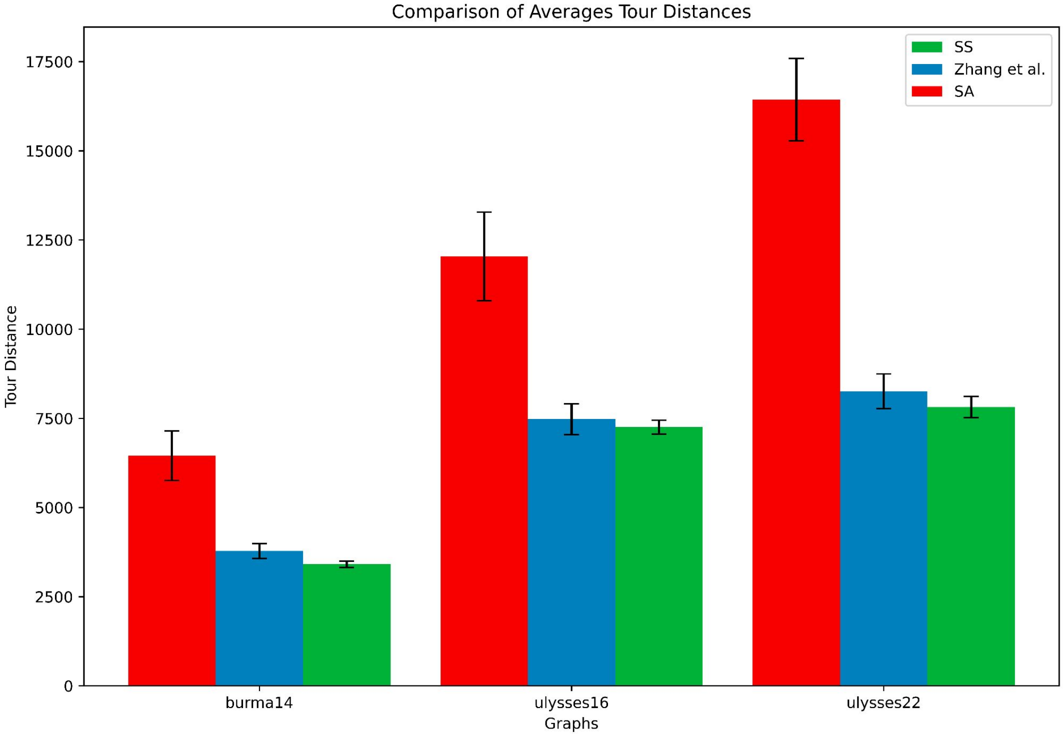
Gmean speedup: 3.3x faster
Max speedup: 2,318.6x faster

This is the fastest implementation we could find.

Goto, H., Endo, K., Suzuki, M., et al. (2021). *High-performance combinatorial optimization based on classical mechanics.*

Results: Traveling Salesman Problem

Comparison of average TSP distances on available graphs



Red: Simulated Annealing
Blue: Zhang et al. (Simulated Bifurcation)
Green: My implementation

Average distance of TSP Tour. Lower is better.

All times: < 1 second
No time provided by other studies

Zhang, T., & Han, J. (2022). *Efficient Traveling Salesman Problem Solvers using the Ising Model with Simulated Bifurcation.*

Conclusions

This is ***the fastest*** Ising solver

Our algorithm is:

- On average ~3x faster…
- And up to ~2,000x faster than the previous leading implementation
- Open-source and free
- Adaptable to any combinatorial optimization problem
- 1000s of “agents” can be run simultaneously

Software Pricing Details

SQBM+ for AWS Learn & Development Plan (Hourly) — **$200.00 /hr** ›
running on p3.2xlarge

Leading implementation costs $200 per hour for use on Amazon Web Services

Access to our algorithm is ***free***

Special Thanks To:

- Dr. Slava Gerovitch & Prof. Srini Devadas

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- Axel Feldmann