Srinivas Arun

Geography, Kotzig's Nim, and Variants

under the direction of Joshua Messing

Abstract

Geography is a combinatorial game in which two players take turns moving a token along edges of a directed graph and deleting the vertex they came from. We study Kotzig's Nim, a special case of Geography where the vertices are labeled and moves correspond to additions by fixed amounts. We prove a conjecture by Fraenkel that Kotzig's Nim is eventually periodic in the size of the graph. We also expand upon work by Fox and Geissler, who classified the computational complexity of determining the winner of various Geography variants given a graph. In particular, we show NP-hardness for undirected partizan Geography with free deletion on bipartite graphs and directed partizan Geography with free deletion on acyclic graphs.

Alan Bu

Concatenations of Two Incidence Matrices, Spanning Trees in Planar Graphs: Two Related Problems

under the direction of Yuchong Pan

Abstract

We call a matrix an *incidence submatrix* if each row has at most one 1 and at most one -1. In this paper, we establish an interesting connection between the concatenation of two incidence submatrices and spanning trees in a planar multigraph, namely that the maximum determinant of such an $m \times m$ concatenation is at least the maximum number of spanning trees in a planar multigraph with m edges. In addition, we present some evidence supporting our conjecture that these two quantities might indeed be equal. This includes several algebraic results on nonplanar graphs. Finally, we give several nontrivial asymptotic lower and upper bounds on these two quantities, showing that they both grow exponentially in m.

David Dong

Comparisons of Methods in Near-to-Far-Field Transformations for Waveguides

under the direction of Rordrigo Arrieta

Abstract

A fundamental problem in exterior scattering theory is to determine the most accurate and computationally simple method to model the scattered field produced by an incoming electromagnetic wave that diffracts off a surface like a perfect electrical conductor. This paper focuses on improving the accuracy of the underlying methods specifically on unbounded scatters, by evaluating the field produced by every point on an imaginary surface. We explore two new methods to accurately compute the electric scatter resulting from these individual points. The first method utilizes the recently developed windowed Green's function, and the second method uses a perfectly matched layer on the same infinite imaginary surface. We analyze, both numerically and through asymptotics, the errors resulting from these two methods under fixed conditions.

Deyan Hadzhi-Manich

On Longest Geometrically Increasing Sequences

under the direction of Yuchong Pan

Abstract

We consider a problem that comes up in the analysis of the *discrete Newton's algorithm for submodular line optimization*. It has to deal with finding *longest geometrically increasing sequences*, that is finding the longest sequence of partial sums of the elements of a real vector $\mathbf{a}$ with n entries, such that each subsequent partial sum is at least twice as large as the previous one. It is known that if we restrict $\mathbf{a}$ to contain nonnegative components only, then such a sequence can contain at most n subsets, and that the maximum length is $\frac{1}{2}n\log_2 n \pm O(n\log\log n)$ in the general case. This result leads us to consider how the number of negative components in $\mathbf{a}$ affects the longest geometrically increasing sequences. We define $G(n,k)$ to be the largest $m \in \mathbb{N}$ such that there exist $\mathbf{a} \in \mathbb{R}^n$ with exactly k negative components and $T_1, \ldots, T_m \subseteq [n]$ for which $\mathbf{a}(T_1), \ldots, \mathbf{a}(T_m)$ form a geometrically increasing sequence. In this paper, we prove several bounds on $G(n,1)$, $G(n,n-1)$ and $G(n,n/2)$, as well as several inequalities between different values of $G(n,k)$. In addition, we report results from a computational study in which the vector $\mathbf{a}$ is drawn from a normal distribution, implying the conjecture that the maximum length in this case grows logarithmically.

Michelle Kang

Bounding the Diameter of Flip Graphs of Split Chessboards

under the direction of Tristan Yang

Abstract

This paper presents an investigation into the diameter of flip graphs associated with split chessboards, which are grid-like structures where unit squares can be separated by a diagonal and adjacent sections are of opposite colors. We conduct an exploration of the properties of flip graphs concerning the standard chessboard and identify distinct connected components within. We further establish a compelling link between split chessboards and plabic graphs, shedding light on essential aspects such as the arrangement of colors on the grid, as well as trip permutations within the disk.

Yifan Kang

CNOT-Optimal Circuit Synthesis

under the direction of Henry Ma

Abstract

The Controlled-NOT (CNOT) gate is central to quantum circuit design due to its role in entanglement generation and information processing. The task of achieving CNOT-optimality becomes increasingly intricate when introducing additional connectivity limitations that restrict the permissible connections between qubits, which can be represented as a graph.

We first give a polynomial-time algorithm for checking if an n-qubit circuit is constructible on a given topological constraint. Our second result is an NP-hardness proof for the problem of finding CNOT-optimal circuits under directed topological constraints. We also make progress towards proving NP-hardness for the analogous problem which considers undirected constraints, which is relevant in real-life settings.

Aaron Kim

The Classification and the Hilbert Polynomials of the Coloring of Quandles with size 6

under the direction of Tristan Yang

Abstract

A *quandle* is an algebraic object that is used to define the colorings of knots because quandles axioms ensures that the number of coloring remain invariant. Schlank and Davis showed that the average number of colorings is asymptotically a polynomial, which is the *Hilbert Polynomial*. They also demonstrated the behavior of quandles of up to size 4 and computed the Hilbert Polynomial. We further examine the computations of quandles of size 6 by generalizing constructions of quandles of size 6 except for few, and use the decompositions to compute the Hilbert Polynomial.

Victoria Paraoulaki de Miranda

Counting the Maximum Number of 3-rich Curves in Configurations of n Points in $\mathbb{P}^2_{\mathbb{F}_q}$

under the direction of Jose Luis Guzman

Abstract

In this paper, we study the problem of counting 3-rich conics in $\mathbb{P}^2_{\mathbb{F}_q}$. A 3-rich conic is a conic that passes through exactly three points in a given set of n points. We consider points in general position, i.e., no three points are collinear and no six points lie on a conic. We explore the maximum and minimum number of 3-rich conics that can be drawn through the configuration of n points in $\mathbb{P}^2_{\mathbb{F}_q}$. First, we establish a lower bound for the number of 3-rich conics, and find an upper bound using combinatorial arguments. We then generalize the upper bound for 3-rich curves of any degree. The total number of 3-rich conics is obtained by subtracting the non-smooth conics (formed by the union of two lines) from the total number of conics. We provide explicit formulas for the number of smooth 3-rich conics generated by the set of n points in general position.

Joseph Vulakh

Positive Traces on Deformations of Kleinian Singularities of Type D

under the direction of Daniil Kliuev

Abstract

Filtered deformations of Kleinian singularities have received much attention over the last half a century, particularly for their importance to various areas of algebra such as representation theory and Lie theory. Recently, traces on filtered deformations of Kleinian singularities have been studied for their connection with star-products, associative products with significance in algebra and theoretical physics, namely superconformal field theory. We build on a line of work investigating positive traces on deformations of Kleinian singularities. In a special case, we find analogues of classification theorems of traces on deformations of Kleinian singularities of type A for Kleinian singularities of type D and prove that positive traces of type D are restrictions of positive traces of type A.

Hongzhou (Hazel) Wu

Classifying bipolynomial Hopf algebras over graded local rings

under the direction of David Jongwon Lee

Abstract

Hopf algebras have been a very important area of research for much of the past century, with people observing and studying such structures in a wide range of fields. Ravenel and Wilson proved that certain bipolynomial Hopf algebras are isomorphic to the Witt Hopf algebra W_R, but only when the underlying rings are $R = \mathbb{Z}_{(p)}$ and $R = \mathbb{F}_p$. We generalize this isomorphism over graded local rings, which creates new possibilities in algebraic topology and other areas of mathematics.

Shu-Ching Yang

Existence of Trapped Vibration Modes in One-Dimensional Crystal Lattices

under the direction of Rodrigo Arrieta Candia

Abstract

This paper presents a rigorous proof of the existence of trapped vibration modes in a one-dimensional monatomic crystal represented as a lattice with masses and strings, subjected to perturbations. By applying the Min-max theorem, we demonstrate that modifying any number of masses, resulting in a decrease in their overall sum, leads to the emergence of at least one trapped vibration mode. We also extend our analysis to general cases where string constants can also be altered and show that as long as the net sum of elastic constants increases, trapped vibration modes still exist. These findings provide valuable insights into the dynamic behavior of perturbed lattices and hold potential applications in diverse fields.

Hanming Ye

The Stable Picard Groups of the Exterior Algebras $E(n)$

under the direction of David Jongwoon Lee

Abstract

Let $E(n)$ denote the exterior algebra on $n+1$ generators. It is a subalgebra of the dual Steenrod algebra. We are motivated to study modules over this algebra because they provide the E_2 page for the Adams spectral sequence, which is used to compute stable homotopy groups. In 1976, Adams and Priddy introduced the *stable Picard group* of modules over $E(n)$, denoted $\mathrm{StPic}(E(n))$. They conjectured that $\mathrm{StPic}(E(n)) \cong \mathbb{Z} \oplus \mathbb{Z}$ for all n. We prove that this conjecture can be reduced to checking only the case $\mathrm{StPic}(E(2)) \cong \mathbb{Z} \oplus \mathbb{Z}$.