2008 SPUR ABSTRACTS

1. Aleksandr Ankhipov

Title: **On The Pairwise Sums Problem**
Mentor: Zuoqin Wang
Abstract: We investigate to what extent a multiset is determined by the multiset of all sums of pairs of its elements. Using generating functions, we characterize all pairs of multisets with the same pairwise sums. In particular, we show that this determination is unique unless the multiset has size 2k, and the multiset of its pairwise sums is symmetric. Additionally, if k is even, there are at most two preimages, and if k is odd, the preimages are symmetric. These properties give us a full characterization for small k.

2. Thomas Belulovich

Title: **Three Problems in Morse Theory**
Mentor: Nikhil Savale
Abstract: A theorem by Franks provides an isomorphism between framed cobordism classes of inclusions and the homotopy types of relative attaching maps in the cellular decomposition of a manifold. First, this theorem is used to compute the homotopy class of the relative attaching maps of the complex Grassmannian manifold.

Secondly, the Morse chain complexes of a manifold with boundary are used to propose an analog to the Mayer-Vietoris sequence in singular homology.

Thirdly, we describe Fukaya's A^∞ category structure on Morse chains and speculate as to possible generalizations to deformed de Rham complexes.

3. Wes Brown

Title: **Antimagic Labelings and the Antimagic Strength of Graphs**
Mentor: Hoda Bidkhori
Abstract: An irregular weighting, or partially weakly antimagic labeling, of a graph G is an edge labeling $L : E(G) \to [t]$ such that the sums of labels of edges incident to each vertex are pairwise distinct. A partially antimagic labeling is an irregular weighting in which all labels are distinct, and a partially antimagic labeling with $t = |E(G)|$ is called an antimagic labeling. The antimagic strength $am(G)$ of a graph G is the smallest t such that a partially antimagic labeling exists.

In this paper, we study antimagic labelings and irregular weightings of different classes of graphs. We prove that (rooted) trees of maximum depth 2 and strict n-ary trees are antimagic. We also find an upper bound for $am(G)$ for trees G with no vertices of degree 2. In addition, we present our own proofs for some known results.

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4. Peter Caday

Title: **Investigations into the Original and Dual McKay Correspondences**
Mentor: Fucheng Tan
Abstract: Let G be a finite subgroup of SU_2. The McKay correspondence is a one-to-one map between the irreducible characters of G and the vertices of an affine Dynkin diagram $\tilde{\Delta}$. A dual correspondence, due to Ito-Reid, relates the conjugacy classes of G to the vertices of the corresponding nonaffine diagram. We define a natural, linear mapping from the irreducible characters of G to complex vectors, and prove that a subset of these vectors form a base for a root system, verifying the McKay correspondence. We also propose a relation between the original and dual McKay correspondences, inspired by a formula of Brylinski, and prove it in part.

5. Rishi Gupta

Title: **Computing the Zeta Function of Elliptic and Hyperelliptic Curves**
Mentor: Ruochuan Liu
Abstract: The Zeta Functions of (hyper)elliptic curves over finite fields store information about the number of points on the curves. This paper introduces a method of computing the zeta function using p-adic cohomology and the Lefschetz fixed point theorem, originally formulated by Prof. Kiran Kedlaya.

6. Sungyoon Kim

Title: **On Dual Stable Grothendieck Polynomials**
Mentor: Hoda Bidkhori
Abstract: The dual stable Grothendieck polynomials are nonhomogeneous symmetric polynomials, which form a basis for the ring of symmetric functions. In this paper, we show that they are closely related to the theory of Schur functions and they have many similar properties as the Schur functions. We generalize the results of Lam and Pylyavskyy, Lenart, and Stanley. Our main results can be expressed as follows:

- We extend the Schur decomposition theorem of Lam and Pylyavskyy to skew shapes.
- As applications, we give a generating function of valued-set tableaux of skew shapes and a simple formula for dual Grothendieck polynomials of border strips.
- We find an expression of a Schur function of skew shape as a sum of dual stable Grothendieck polynomial with coefficients which can be expressed combinatorially.
- We study the relation between the number of elegant fillings and the number of strict elegant fillings.
- We express a skew shape dual stable Grothendieck polynomial as a sum of dual stable Grothendieck polynomials, and we study the coefficients.

7. Yuncheng Lin

Title: **Szemerédi-Type Problems in Finite Fields**
Mentor: Julia Wolf
Abstract: A central problem in arithmetic combinatorics is to determine the maximum size of a subset of an additive group G that contains no k-term arithmetic

progressions. We denote the maximum size of such a set by $r_k(G)$. In this paper we give an exposition of the best known upper and lower bounds on $r_3(G)$ in the case when $G = \mathbb{F}_3^n$. We also prove that for any fixed integer k and any prime power $q > k$, there exists a subset of $\mathbb{F}_q^k$ of size q^{k-1} which contains no k points on a line, and hence no k-term arithmetic progressions. As a corollary and main original result of this paper we obtain an asymptotic lower bound for $r_k(\mathbb{F}_q^n)$ when $q > k$ as $n \to \infty$.

8. Cordelia Link

Title: **On Sets Whose Difference Set Contains No Squares**
Mentor: Julia Wolf
Abstract: This paper concerns a number of questions one can ask about subsets of $[N]$ and $\mathbb{Z}/p\mathbb{Z}$ that contain no square differences. The best known lower bound for a square difference free subset of $[N]$ was given by Beigel and Gasarch. This bound is a numerical improvement of a construction given by Ruzsa. We discuss various strategies to numerically improve this bound. While our efforts have not born fruit at the time of writing, we have discovered a numerical improvement for subsets of $[N]$ which contain no cube differences. We also adapt Ruzsa's construction to a function field setting, complementing Le Thai's previous construction of an upper bound in this setting. Finally, we examine the corresponding coloring problem. We present a lower bound and an exact asymptotic formula for the number of monochromatic square differences in a 2-coloring of $[N]$ and $\mathbb{Z}/p\mathbb{Z}$ respectively.

9. Yinghui Wang

Title: **Some Study On The Generalized Euler-Maclaurin Formula**
Mentor: Zuoqin Wang
Abstract: In this paper, we will study the generalized Euler-Maclaurin formula, which is used to calculate $\frac{1}{N^n} \sum_{k \in N\Delta \cap \mathbb{Z}^n} f(\frac{k}{N})$, for non-simple regular wedges W (to be be defined in the paper) and for "curved" wedges or polytopes defined by polynomials with integer coefficients. We will be using the exclusion-inclusion method thoughout the calculations and proofs in this paper. By studying the two-dimensional case first, we will obtain two interesting identities of the Bernoulli numbers and roots of unity. For the two-dimensional or higher-dimensional cases, we will also proof a simplified formula for a non-simple regular polytope composed of two simple ones. At last we will derive a formula for a particular kind of "curved" wedges and polytopes, under some restrictions on the given function f.

10. Bohua Zhan

Title: **Representing Graphs by Linear Subspaces**
Mentor: Ruochuan Liu
Abstract: We study representations of graphs by assigning vertices to linear subspaces with the condition that two subspaces intersect if and only if the corresponding vertices are connected. We consider the problem in both affine and projective geometries. We prove several results relating representability in different dimensions, construct representations for general graphs, and construct graphs that are not representable for a given dimension.