

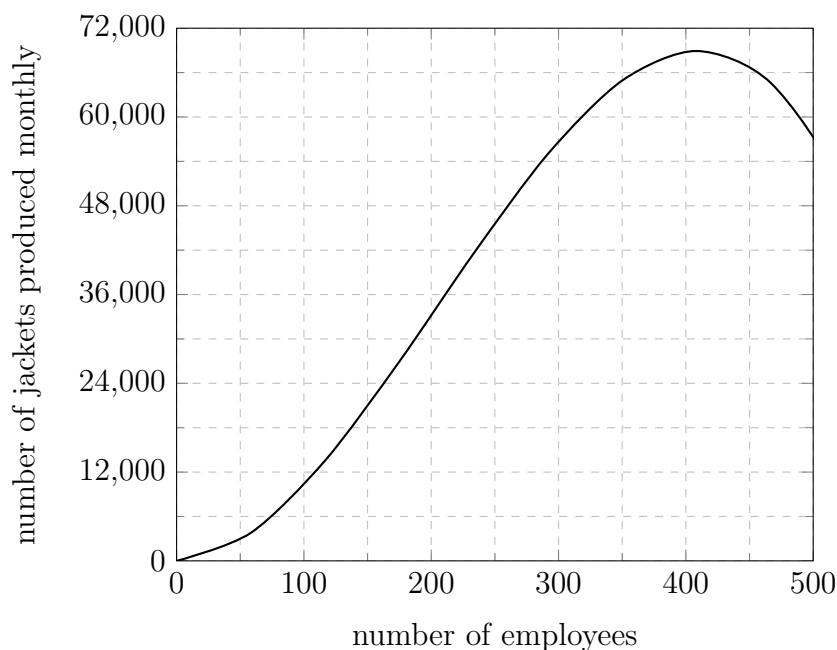
Name: _____

Section: _____

Collaborators: _____

Lab 9: The Law of Diminishing Returns

A company producing hand-detailed jackets finds that the number of jackets produced each month depends on the number of employees working in production according to the function $f(x)$ graphed below, where x is the number of employees, and $f(x)$ is the number of jackets produced each month.



1. From the graph, approximate the value of each of the following derivatives.
 - $f'(150)$
 - $f'(350)$
2. For each of the derivatives you estimated in problem 1, interpret the meaning of that value in terms of employees and monthly production:
 - Interpret $f'(150)$:

- Interpret $f'(350)$:

3. Using the graph, fill in the table below indicating whether $f'(x)$ and $f''(x)$ are *positive* or *negative* at the indicated values of x :

Value of x	$f'(x)$: positive or negative?	$f''(x)$: positive or negative?
100		
300		
500		

4. Using the graph, approximate the following:

- Critical point(s) of $f(x)$:
- Interval(s) where $f'(x)$ is positive:
- Interval(s) where $f'(x)$ is negative:

5. Interpret what it means for $f'(x)$ to be positive, in terms of employees and monthly production.

6. Interpret what it means for $f'(x)$ to be negative, in terms of employees and monthly production.

7. Does it make sense for this company to hire 500 employees to work in production? Why or why not? In order to maximize monthly production, approximately how many employees should they hire?

8. Using the graph, approximate the following:

- Interval(s) where $f'(x)$ is increasing:
- Interval(s) where $f'(x)$ is decreasing:
- Interval(s) where $f(x)$ is concave up:
- Interval(s) where $f(x)$ is concave down:
- Inflection point(s) of $f(x)$:

9. Interpret what it means for $f'(x)$ to be increasing, in terms of employees and monthly production.

10. Interpret what it means for $f'(x)$ to be decreasing, in terms of employees and monthly production.

In a graph such as this, economists call the inflection point the *point of diminishing returns*. Starting at the point of diminishing returns, if the number of workers is increased, while total monthly production may continue to rise, the increase in production per additional worker is decreasing.

15. Find the derivative $f'(x)$.
16. Using the derivative, find the number of employees that maximizes monthly production.
17. Find the second derivative $f''(x)$.
18. Find all possible points of inflection for f by setting $f''(x) = 0$ and solving for x .
19. Using the second derivative, find the following:
- Interval(s) where $f(x)$ is concave up:
 - Interval(s) where $f(x)$ is concave down:
 - Inflection point(s) of $f(x)$:
20. Where is the point of diminishing returns for $f(x)$?

21. A company that grows and sells arugula is interested in how fertilizer use affects their arugula production. They observed the following data:

Pounds of fertilizer per acre	40	80	120	160	200	240	280	320
Yield of arugula per acre	6000	6500	7200	8000	8600	9000	8750	8400

- Using the table, approximate the pounds of fertilizer per acre that the company should use to maximize arugula production.
- Using the table, approximate the point of diminishing returns.
- Interpret what the point of diminishing returns means in this situation.

22. For a function $g(x)$, fill in the following table illustrating the relationship between $g(x)$, $g'(x)$, and $g''(x)$. For each box in the table fill it with the appropriate selection from the following choices: *negative, positive, zero, decreasing, increasing, level, concave down, concave up, neither concave up nor concave down*.

If $g''(x)$ is:	Then $g(x)$ is:	Then $g'(x)$ is:
Positive		
Negative		
Zero		

Participation	/5
Correctness	/5
Total	/10