Inquiry and Engagement in an interactive classroom

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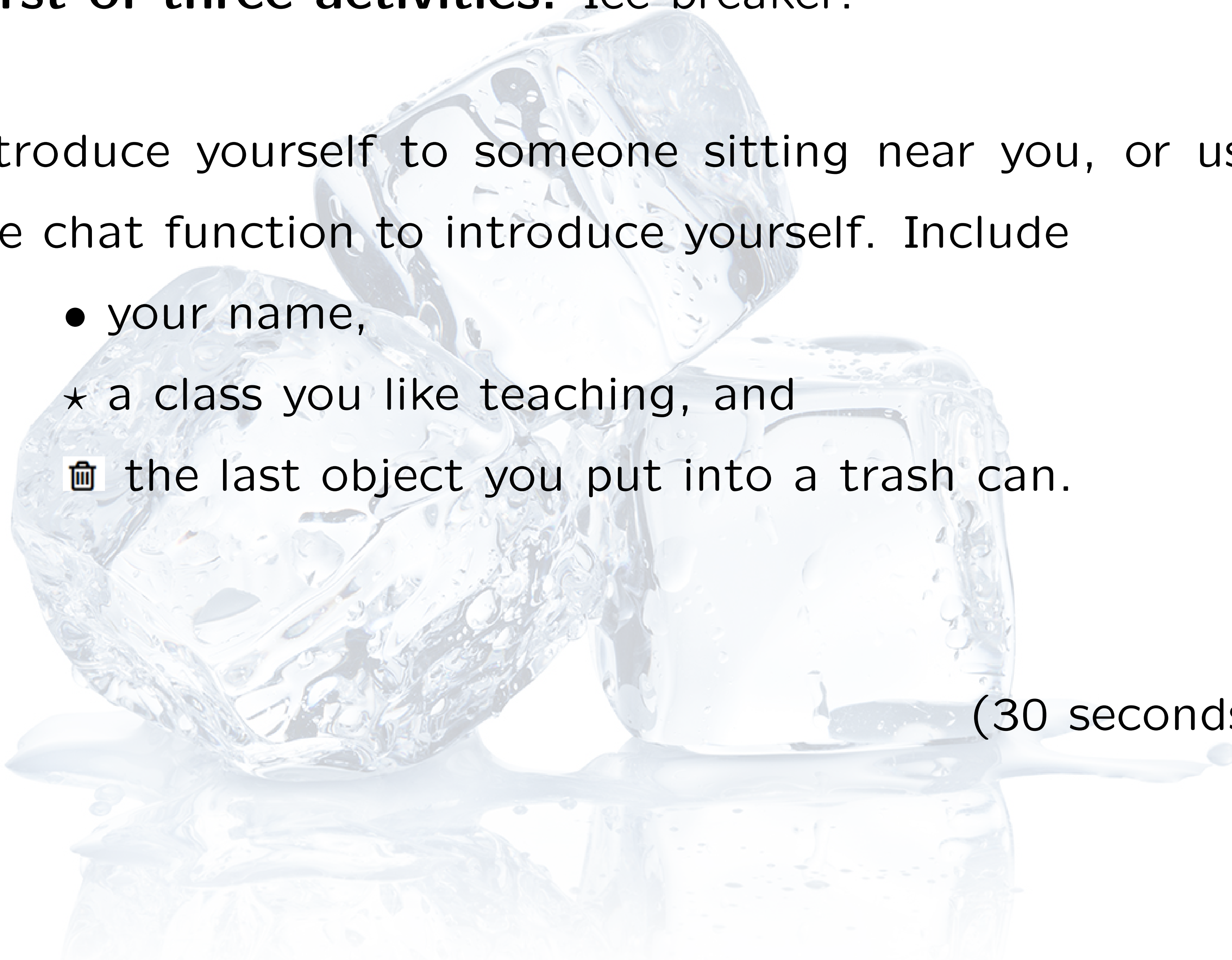
Franklin & Marshall College

2019

Warning!

In this talk, I have included three short activities for you and people around you.



First of three activities: Ice breaker.

Introduce yourself to someone sitting near you, or use the chat function to introduce yourself. Include

- your name,
- ⋆ a class you like teaching, and
- 🗑 the last object you put into a trash can.

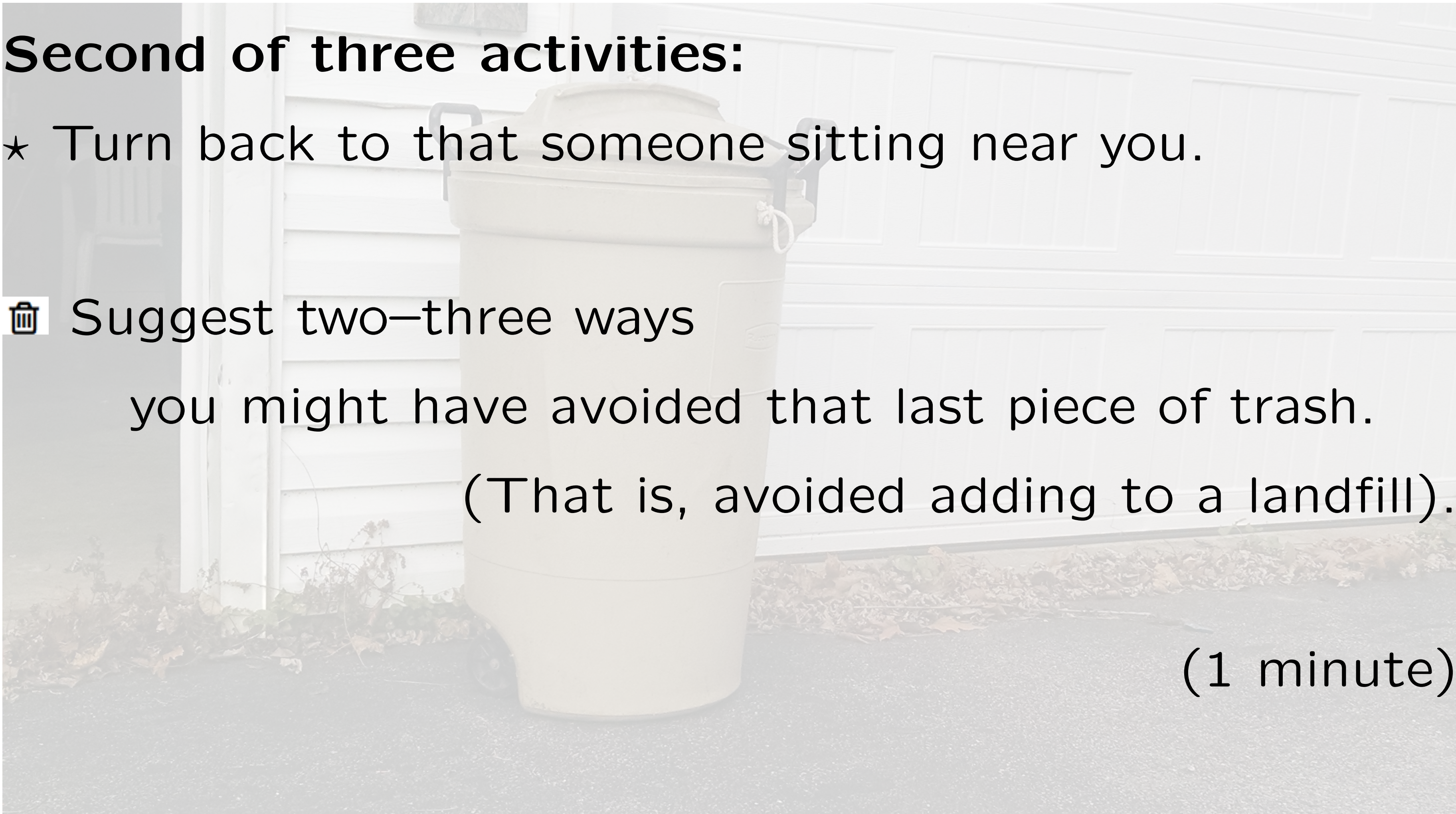
(30 seconds)

(Time's up.)

Thank you!!!

In 2018, my family (with 3–6 people at home)

put 6 trashcans at the curb.



Second of three activities:

⋆ Turn back to that someone sitting near you.

Suggest two–three ways

you might have avoided that last piece of trash.

(That is, avoided adding to a landfill).

(1 minute)

(Time's up.)

Thank you again!!!

Your responses probably fell into one of these six categories:

1. R__f__s__ the trash in the first place
2. R__pl__c__ with a trash-free alternative
3. R__ __s__ durable objects vs. disposable objects
4. R__d__c__ the amount of things you use
5. R__c__cl__ instead of tossing in the landfill bin
6. R__t organic items

credit: The Zero Waste Home

1. R__f__s__: Avoid the trash in the first place

2. R___pl___c___: Change to a trash-free alternative

versus

3. R__ __s__ durable objects, such as water bottles, bags, spoons, cloth napkins, ‘xeryp’ containers, etc.

4. R__d__c__ the amount of things you use

5. R__c__cl__ instead of tossing in the landfill bin

6. R__t organic items

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As my daughter once told her friends,

"My mom has a PhD *and* a compost pile!"

credit: The Zero Waste Home

Bea Johnson's 2015 trash

So . . .

. . . what does "talking trash" tell us about teaching math?

Talking to strangers is uncomfortable for a lot of people.

Talking to friends makes it easy to get off-track.

The room set-up is bad for effective conversation & follow-up.

We come to a talk (or a class) to hear from an "expert", not from the random person sitting next to us.

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So make the rules specific and explicit:

timing, roles, and scope of discusssion.

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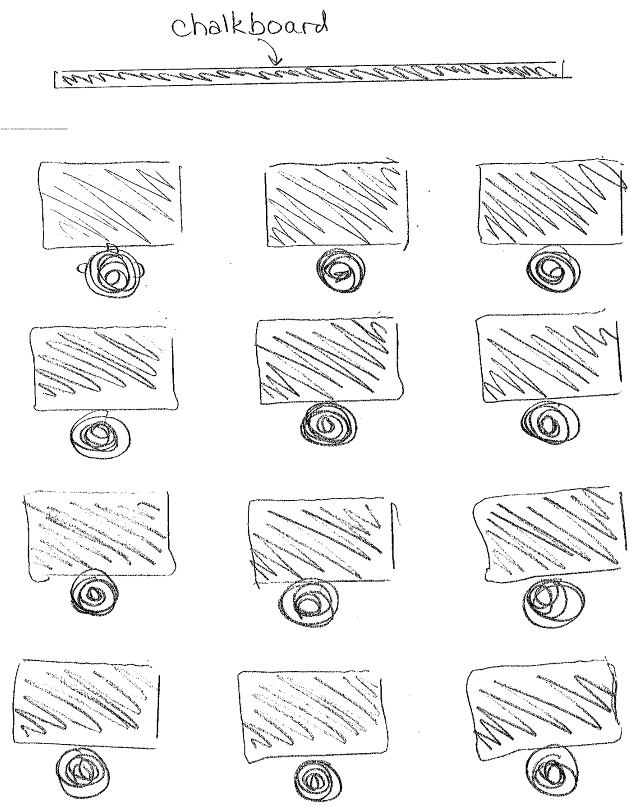

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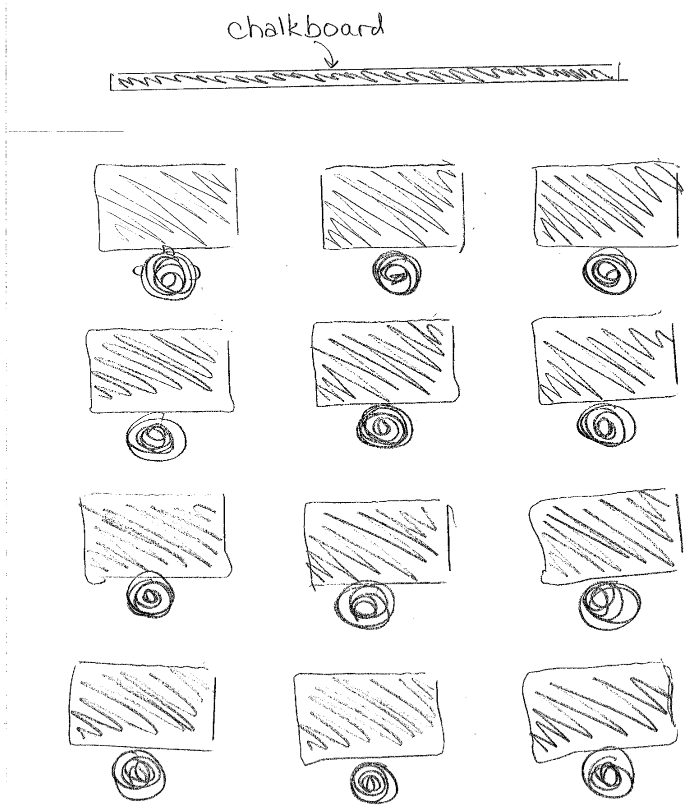

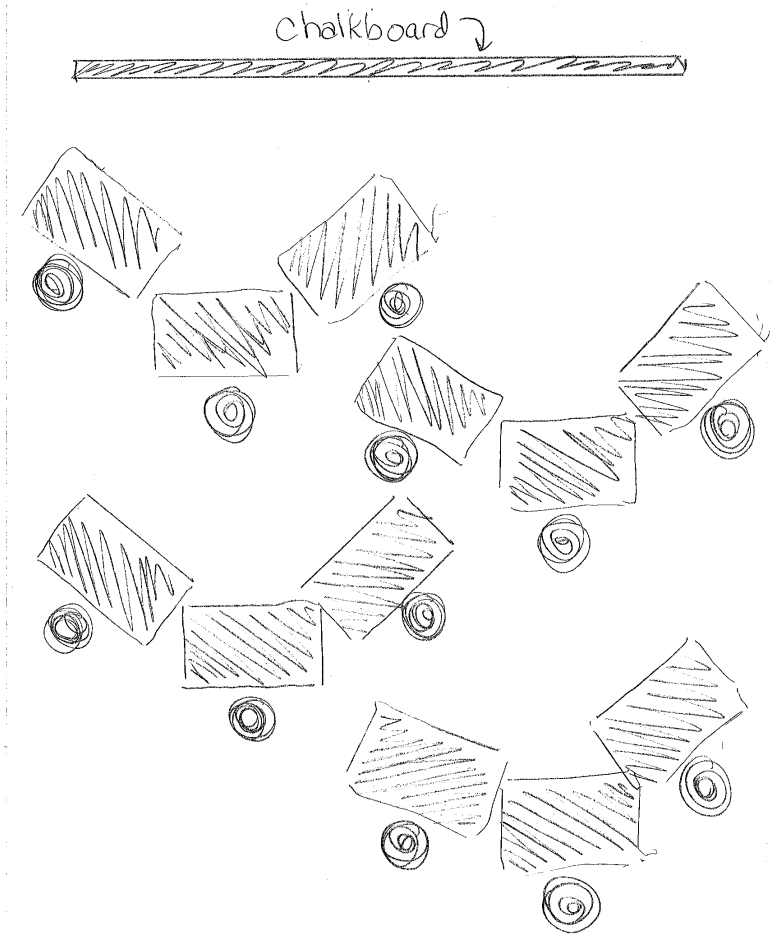

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to hear from an "expert",

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Answer 1, for the instructor:
So carefully craft questions that people can solve themselves, *with some difficulty*, together.

We come to a talk (or a class)
to hear from an "expert",
not from the random person sitting next to us.

Answer 2, for the student:

But what we want, and what help us learn best,
are two different things.

What helps us learn best? *Interactive Engagement*

What helps us learn best? Interactive engagement has three components for students:

1. struggle with the problem themselves;
2. talk to other students about their approaches;

and then

3. get immediate feedback.

see Epstein, *AMS Notices* (2013).

Interactive Engagement includes good, reasonable struggle.

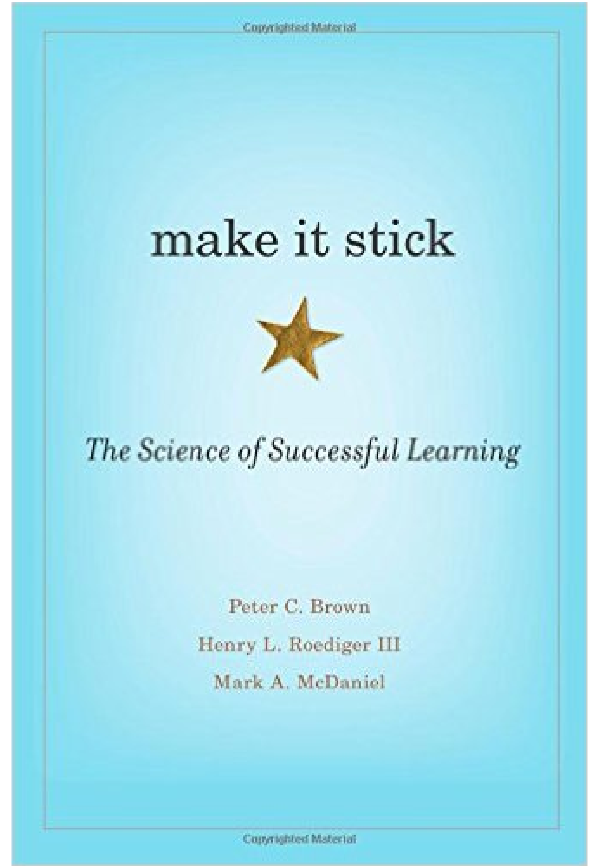
We prefer the solutions we come up with ourselves, not the ones other people give us. So introduce “desirable difficulties”:

1. R__f__s__ the trash in the first place.

vs.

1. Refuse the trash in the first place.

It is better to guess a wrong answer (and get feedback) than to make no attempt. When learners commit errors and are given feedback, the errors are not learned.



Brown, Roediger, and McDaniel (2014)

What does it mean to say *Interactive Engagement* "works better"?

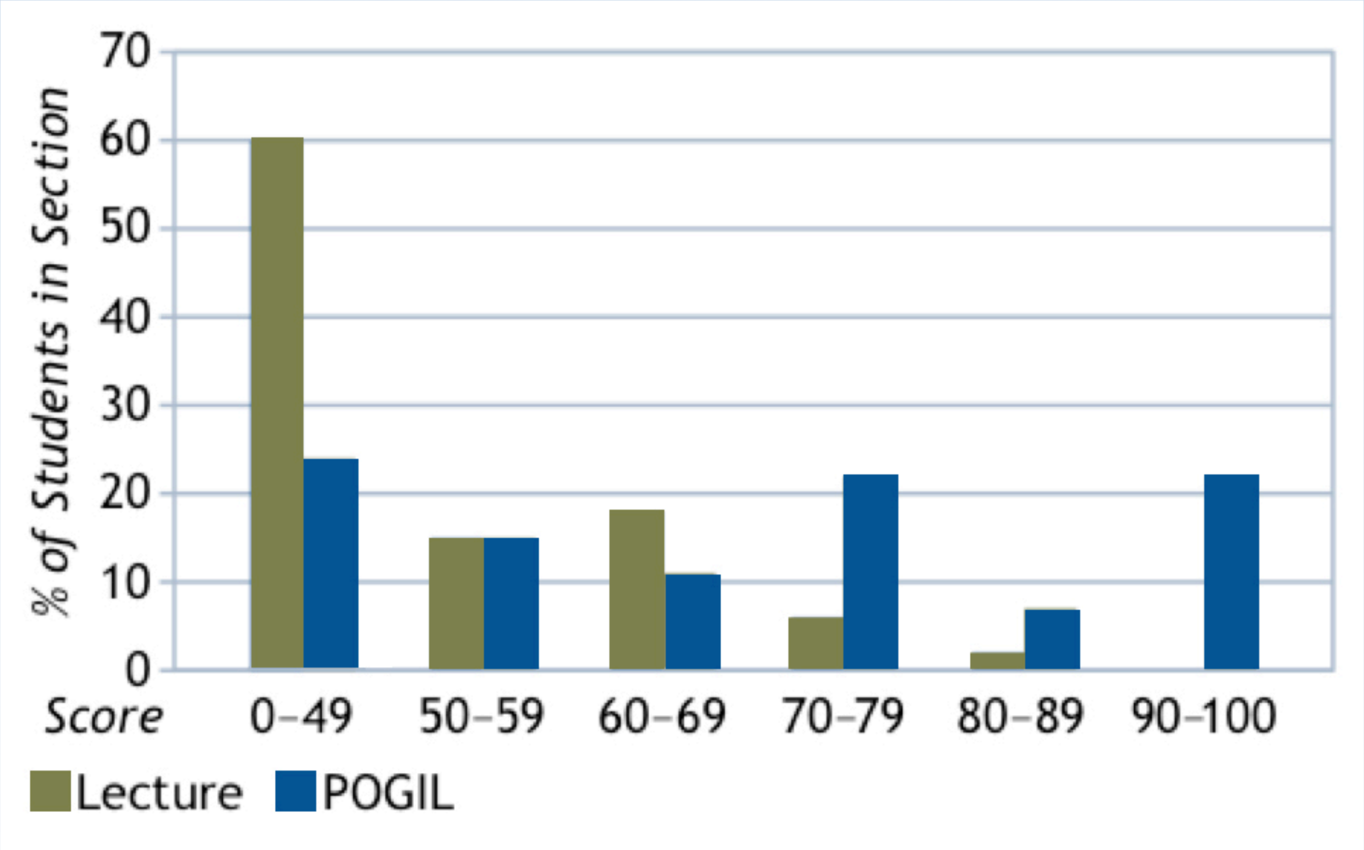
Next page:
Organic Chem II student grades
on a "review" test covering Organic Chem I material.

Gray/green bars: lecture,

Blue bars: Inquiry/discovery method

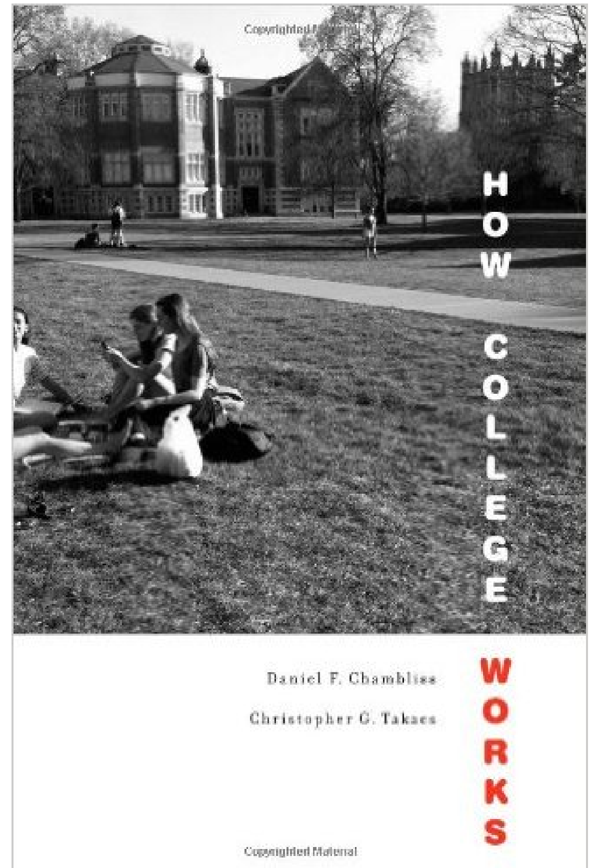
Organic Chemistry 2 Pre-Quiz at a Large Public University

Organic 2 Pre-quiz Results (Lecture vs. POGIL Organic 1)



Ruder, S.M., & Hunnicutt, S.S. (2008). POGIL in Chemistry Courses at a Large Urban University: A Case Study. In R.S. Moog, & J.N. Spencer (Eds.), *Process-Oriented Guided Inquiry Learning: ACS Symposium Series 994* (pp. 133–147). Washington, D.C.: American Chemical Society.

"College works when it provides a thick environment of constant feedback, driven by the establishment and maintenance of social relationships."



Chambliss and Takacs, 2014.

My first (accidental) IBL class:

I was jealous of the Geology Department, so I took my students to a coffee house.


Don't limit a child to your own learning, for he was born in another time
I have measured out my life with coffee spoons
T.S. Eliot


Los Compadres
PUSH

My first worksheets were masterpieces in pencil.

SQUARE ONE COFFEE HOUSE APRIL 2, 2002

INVESTIGATING COSETS & NORMAL SUBGROUPS

EXAMPLE 1

PULL OUT YOUR HANDY-DANDY CAYLEY TABLE FOR D_3.

WE WILL INVESTIGATE LEFT & RIGHT COSETS OF 2 SUBGROUPS:

$\mathcal{R} = \{0, 1, -1\}$ and $\mathcal{F} = \{0, V\}$

FILL IN THIS TABLE

a	$a\mathcal{R}$	$\mathcal{R}a$	$a\mathcal{F}$	$\mathcal{F}a$
0				
1				
-1				
V				
P				
N				

$\mathcal{R}$ is a normal subgroup; $\mathcal{F}$ is not. What do you think the definition of a normal subgroup is?

EXAMPLE 2

U_{18}, like D_3, has subgroups of order 3 and 2.

$\mathcal{U}_{18} = \{ \quad \}$

$H = \{ \quad \}$ is the subgroup of order 3

$J = \{ \quad \}$ is a subgroup of order 2.

List all cosets of H in $\mathcal{U}_{18}$:

List all cosets of J in $\mathcal{U}_{18}$:

Is H a normal subgroup of $\mathcal{U}_{18}$?

Is J a normal subgroup of $\mathcal{U}_{18}$?

PAGE 2

(4) $\mathcal{F}$ is isomorphic to a group we already know and love. What is that group? ________

(5) Define a function ϕ which serves as an isomorphism from $\mathcal{F}$ to that other group.

$\phi(\frac{1}{x}) =$ ______

$\phi(1-x) =$ ______

(6) Explain why ϕ is 1-to-1.

(7) Explain why ϕ is onto.

(8) Explain why ϕ preserves group operation.

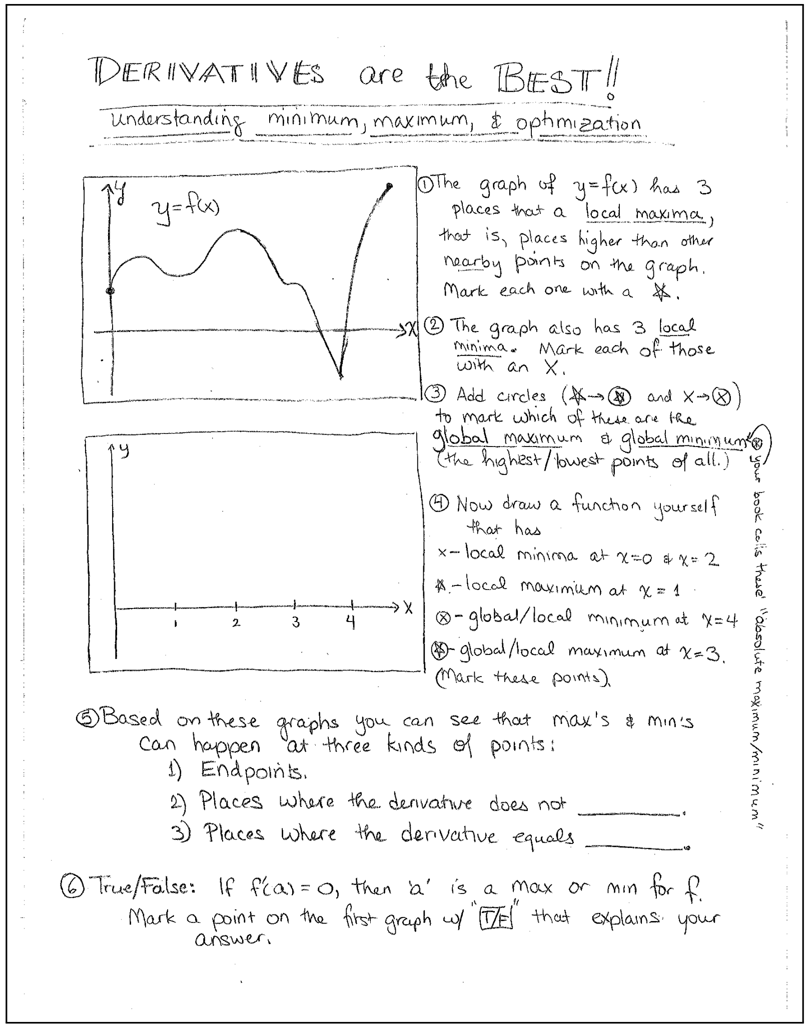
ANNOUNCEMENTS

Come to Job Candidate talks: Friday, Monday, Wednesday!

There is no homework due Tuesday March 19.

There's a Math Club Meeting Wednesday the 20th at 7:30, Stager 212.

I started using occasional worksheets in calculus classes, too.


DERIVATIVES are the BEST!!

understanding minimum, maximum, & optimization

① The graph of $y=f(x)$ has 3 places that a local maxima, that is, places higher than other nearby points on the graph. Mark each one with a ✱.

② The graph also has 3 local minima. Mark each of those with an X.

③ Add circles (✱→⊛ and X→⊗) to mark which of these are the global maximum & global minimum* (the highest/lowest points of all.)

④ Now draw a function yourself that has

x – local minima at $x=0$ & $x=2$

✱ – local maximum at $x=1$

⊗ – global/local minimum at $x=4$

⊛ – global/local maximum at $x=3$.

(Mark these points).

* your book calls these "absolute maximum/minimum"

⑤ Based on these graphs you can see that max's & min's can happen at three kinds of points:

1) Endpoints.

2) Places where the derivative does not ________.

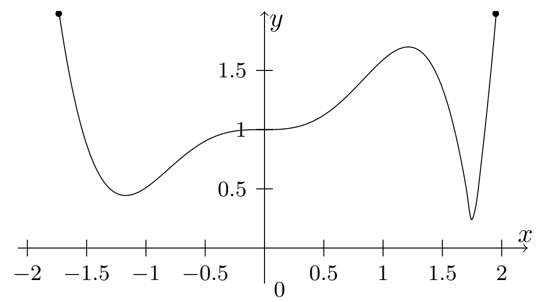
3) Places where the derivative equals ________.

⑥ True/False: If $f'(a)=0$, then 'a' is a max or min for f. Mark a point on the first graph w/ "T/F" that explains your answer.

And then Max came along . . .

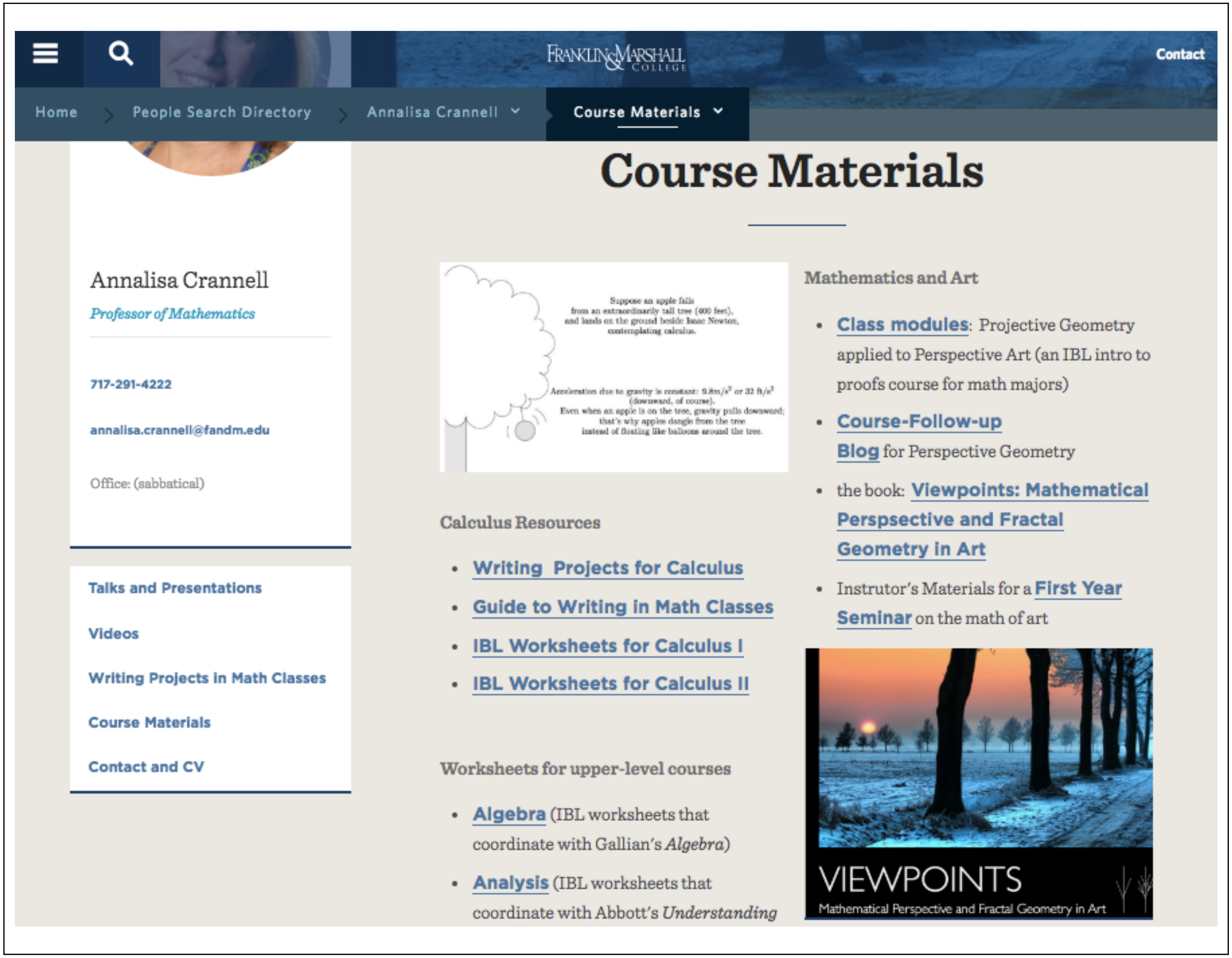
Now I have lots of LaTeX-ed worksheets . . .

Critical Points and Optimization **4.2**



1. The graph above has three local maxima and two local minima. Circle these five points.
2. Then fill in the blanks in the following sentence: If f is a function whose domain is the interval $[a, b]$, then local minima and local maxima happen either
   (a) at the ________________ of the interval, or
   (b) at places where the derivative is equal to ________, or
   (c) at places where the derivative does not ________.
3. [True or False?] If $f'(c) = 0$ for some value $x = c$, then f has a local maximum or minimum at $x = c$.
4. Suppose the volume of a box is given by $V(h) = 180h - 39h^2 + 2h^3$, where the height h of the box ranges from 0 to $\frac{15}{2}$.

. . . that I've posted on my web pages, for the curious



Even more worksheets for IBL math classes:

Journal of Inquiry-Based Learning in Mathematics

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Journal Contents

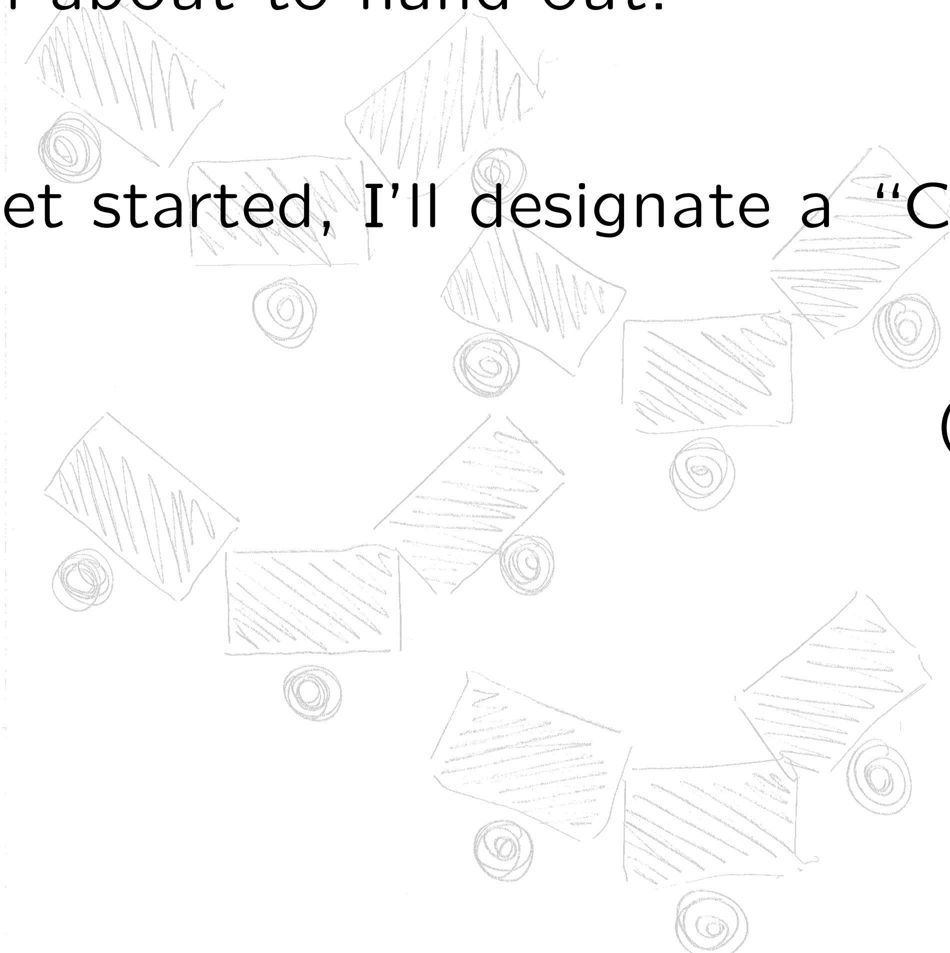
These refereed course notes fulfill the guidelines listed under Information for Authors.

This table is sortable. Click on a column heading to sort by that column. Click on a title to access these notes.

Title	Author	Subject	Pages	Issue
Point-Set Topology	Orloff Clark , Lisa	topology	28	No. 41 (Apr. 2016)
Topology Notes	Person, Laura J.	topology	57	No. 40 (Jan. 2016)
Calculus I: Notes and Problems	Stallmann, Cornelius	calculus	76	No. 39 (Sept. 2015)
Discovering Infinity	Olinick, Michael	set theory	71	No.

Third of three activities:

⋆ Turn your desks into "V"s and do questions 1–3 of the worksheet I'm about to hand out.



⋆ After you get started, I'll designate a "Chalk Czar".

(≈ 5 minutes)

The Game of Gen-Re

The object of the game is to make as many different words as you can with a given set of letters. Extra points goes to the person who can show that there are no more possible words!

> Math major's advantage: Unlike most word games, you don't have to spell. In fact, most of the words you'll create in Gen-Re aren't English words at all!

Playing pieces:

I will give you a set of letters (these letters are called the “generators”). I will also give you a set of equations (called “relations”).

Rules of the game:

1. A word is any string of those letters, in any order, including the 0-letter word "$\{ \}$".

2. Two words are the same if you can use a relation to replace one string of letters by its equivalent string.

3. Transitivity holds for "sameness": two words are the same if we can apply rules 1 and 2 repeatedly to get from one word to the other.

Scoring:

- One point for every word on your list.
- Two points for proving this is the largest possible list of different words.
- Minus three points for every word on your list that is the same as another word on your list.

We'll play Game 3:

($\approx$ 5 minutes)

Game 1
Generators: R;
Relations: RRRRR={ }

Game 2
Generators: N, i;
Relations: ii={ }, NNN={ }, iN=Ni

Game 3
Generators: a, Y;
Relations: aa={ }, YYY={ }, YaY=a

Game 4
Generators: a, e, t;
Relations:
aa=ee=tt={ }, at=ta, ae=ea, et=te

Game 5
Generators: o, B;
Relations:
oo={ }, BB={ }, oB=BoBoBo

(Time's up.)

Thank you again!!!

Implementing IBL

Developing questions

Assigning groups

Structuring the group work

What you do while they're working

Giving feedback

How to develop questions? Two wide-spread approaches to IBL:

- (Modified) Moore Method

- POGIL

  Proccess-**O**riented, **G**uided **I**nquiry **L**earning

Example of a Moore-Methodesque approach,

in Abstract Algebra

Definition. For any element g in a group G with identity e, we say that $|g|$, the *order* of g, is the smallest positive integer n so that $g^n = e$. If such an integer does not exist, we say $|g| = \infty$.

For each of the statements 1–5 below, either prove the statement or find a counter-example.

1. If G contains an element of order 4, then all elements except the identity have order 4.

2. If G contains an element of order 4, then G contains an element of order 1.

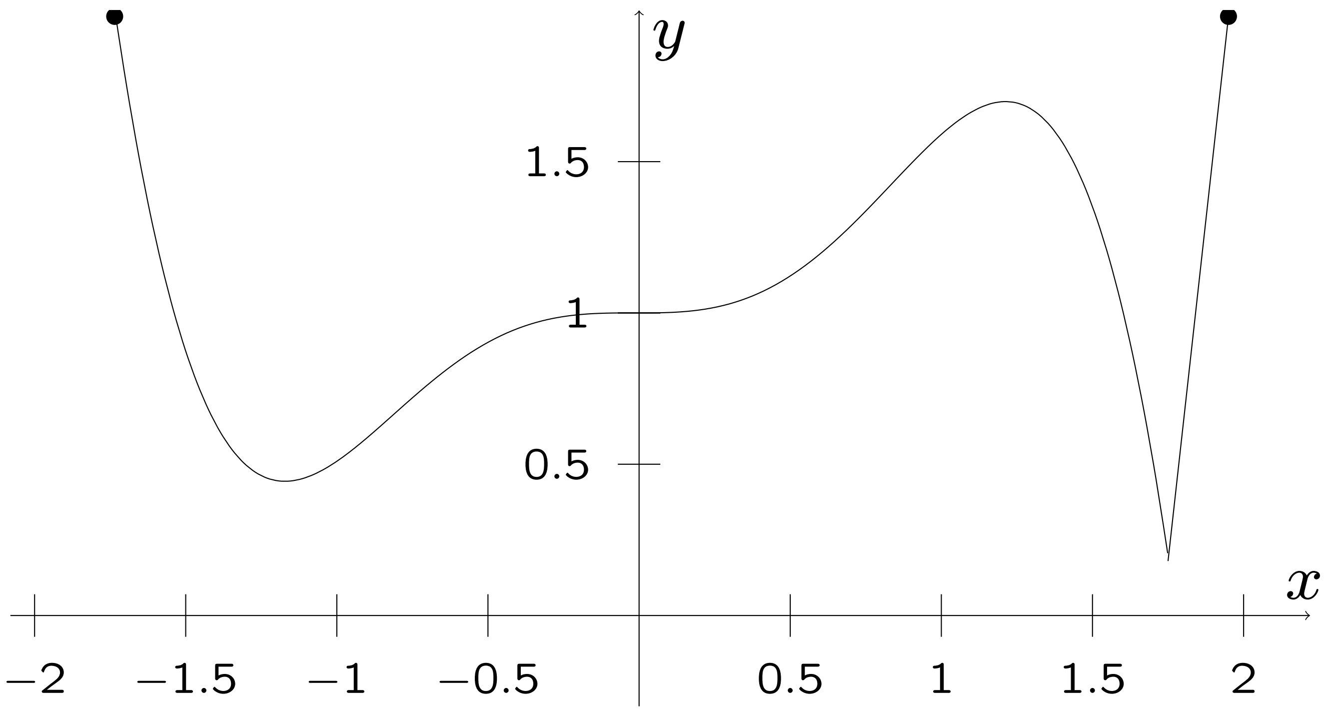
3. If G contains an element of order 4, then G contains an element of order 2.

4. If G contains an element of order 4, then G contains an element of order 3.

5. If G contains an element of order 4, then that is the only element in G with order 4.

Example of a POGIL-esque approach,

in Calculus I



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Then fill in the blanks in the following sentence: If f is a function whose domain is the interval $[a, b]$, then local minima and local maxima happen either

(a) at the ____________________* of the interval, or

(b) at places where the derivative is equal to __________, or

(c) at places where the derivative does not __________.

Values of x that satisfy (b) or (c) above are called *critical points* of f.

* *Hint:* Unscramble the letters "d e i n n o p s t".

How to assign groups?

- with charts
- writing symbols on worksheets
- using questionaires
- let the students decide

(3–4 people per group seems to work well)

Structure the discussion

- Which problems to work on?
- What roles do students have within groups?

  (Chalk Czar!)

- When and how do they stop working?

What the instructor does . . .

- nudge students to answer each other's questions

  "What does your group think?"

  "Sam wants to know if $x = 2$. Do you all agree?"

  "Does everyone in your group have the same answer?"
- assign "ambassadors" from one group to another
- encourage students to take risks

  "Put it up on the board! If it's a mistake,
  it's a good mistake and everyone will learn from it."

Feedback and discussion

- Look at the students, not at the board.
- Ask students who didn't put the answer up to explain it.
- Promote risk-taking and learning from "good" mistakes
- Promote "growth" mind-set and effort

In conclusion . . .

In conclusion . . .

Something witty.

(*talking trash? an IBL mind is a terrible thing to waste?*)

Thank you.

References

Brown, Roediger, and McDaniel, *Make it Stick: the science of successful learning*, The Belknap Press of Harvard University Press: London, England, 2014.

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Annalisa Crannell's classroom materials:
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Jerome Epstein, "The Calculus Concept Inventory—Measurement of the Effect of Teaching Methodology in Mathematics", *AMS Notices* **60**:8 (2013) 1018–1026.

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Bea Johnson, `http://www.zerowastehome.com/`

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