Topology Seminar

Jeremy Hahn

of MIT will be speaking on

Highly connected manifolds in dimensions larger than 248

on September 9 at 4:30 in MIT Room 2-131

I will survey the problem of classifying smooth, (n - 1)-connected, closed (2n)-manifolds, at least in large dimensions 2n > 248. The classification up to diffeomorphism was first attempted in a 1962 paper of C.T.C. Wall, where it was related to questions about the boundaries of "almost closed" manifolds. Several of these questions were answered in the 70s and 80s, most notably by Stephan Stolz, but for example the Kervaire Invariant 1 question remained unresolved until 2009.

I will explain work in progress, joint with Robert Burklund, Tyler Lawson, and Andrew Senger, proving for n > 124 that the boundary of any (n-1)-connected, almost closed (2n)-manifold also bounds a parallelizable manifold. In large dimensions this solves the last of Wall's original questions about his boundary homomorphism, determines all Stein fillable homotopy spheres, and proves a conjecture of Galatius and Randal-Williams.

Work of Galatius, Randal-Williams, and Krannich relates our theorem not just to the classification of (n-1)-connected (2n)-manifolds, but also to the calculation of their mapping class groups.