David Vogan October 2, 2019, *The mysterious KGB: how atlas writes down a group and a little bit more.*

Note that this week's session *only* will take place Wednesday 4:00–5:00 in MIT Room 2-355, at the usual Zoom meeting number
$$\texttt{https://zoom.us/s/739477128}.$$

Recall that `atlas` represents a complex reductive group G as

$$(n, \{\alpha_i \mid 1 \le i \le \ell\}, \{\alpha_i^\vee \mid 1 \le i \le \ell\})$$

with $\alpha_i \in \mathbb{Z}^n$ (thought of as the lattice $X^*(T)$; these are the simple roots), $\alpha_i^\vee \in \mathbb{Z}^n$ (thought of as the lattice $X_*(T)$; there are the simple coroots). The only requirement on these data is that the $\ell \times \ell$ matrix $(\langle \alpha_i, \alpha_j^\vee \rangle)$ should be a Cartan matrix.

Main point is that if G and G' are two *pinned* complex reductive groups with these data (a pinning being a set $\{X_{\alpha_i}\}$ of simple root vectors), then there is a *unique* isomorphism from G to G' carrying each X_{α_i} to X'_{α_i}.

Fix now an "inner class" of real forms of G. This means an involutive automorphism of the based root datum; that is, an $n \times n$ integer matrix δ such that

$$\delta^2 = 1, \qquad \delta(\{\alpha_i \mid 1 \le i \le \ell\}) = \{\alpha_i \mid 1 \le i \le \ell\},$$
$$\delta^t(\{\alpha_i^\vee \mid 1 \le i \le \ell\}) = \{\alpha_i^\vee \mid 1 \le i \le \ell\}.$$

The software lets you look at all these things for a given real form, and we'll do a lot of that. Given a *semisimple* G, there are only finitely many δ satisfying the condition above, and the software will list them; we'll do that.

If G has a center (of rank greater than one) then there are infinitely many possible δ (although only finitely many up to conjugation by $\operatorname{Aut}(G)$). The software doesn't try to list representatives for the δ up to $\operatorname{Aut}(G)$; making a script to do that would be an entertaining and somewhat useful exercise.

The rest of the session Wednesday will look at how the results Timothy talked about last week lead to enumeration of the real forms in an inner class, and how they are recorded in `atlas`. In general I hope the session will be 75% software demo and 15% math. (The other 10% is just margin of error.)