

18.434 Problem Set #2, Fall 2011

The problem set is due on Monday October 24, 2011. Solutions should be typeset. You are welcome to brainstorm with other students in the class; however, you have to write your own solutions in your own words without looking at the write-up of other students' in the class.

1. Let $G_{n,k}$ be the graph with vertex set $V = \{0, 1, 2, \dots, n-1\}$ considered cyclically, and an edge between i and $j \neq i$ if and only if $i - j \pmod{n} \leq k$ (e.g. in $G_{10,2}$ vertex 1 is adjacent to vertices 10, 0, 2, 3). $G_{n,k}$ is a $2k$ -regular graph (every vertex has degree k).
 - (a) What are the eigenvalues of the Laplacian of $G_{n,k}$?
 - (b) For k fixed, how does λ_2 behave as a function of n (just give the leading term as a function of n in $O(\cdot)$ or $\Omega(\cdot)$ notation)? Does the family of graphs $G_{n,k}$ (parametrized by n) constitute an expander family?
 - (c) What is the number of spanning trees of $G_{n,2}$? There is a simple closed-form formula for it. (To guess it, you may want to experiment with matlab for a few values of n . To prove it, you may want to try to derive a recurrence relation.)