ERRATA AND ADDENDA

to

*Enumerative Combinatorics*, volume 1, second printing

by

Richard P. Stanley

(version of 24 November 2021)

• p. 6, line 5–. Change compositon to composition.

• p. 11, Example 1.1.16, line 5. Change $Y_i$ to $Y_j$ (twice) and $X_i$ to $X_j$.

• p. 16, line 3–. Change $x_i$ to $y_i$.

• p. 19, line 19–. Change $[k]$ to $[n-k]$. This correction needs to be made in the hardcover but not the paperback edition.

• p. 19, line 4–. This should say $0 \leq a_i \leq x + n - i - 1$. It is correct in the hardcover edition and incorrect in the paperback edition.

• p. 19, line 7–. Change sufficies to suffices.

• p. 20, line 7–. It would be more accurate to replace “The proof of Proposition 1.3.7” with “The third proof of Proposition 1.3.4”.

• p. 24, Proposition 1.3.14, part 3, line 2. Replace “$k$ vertices” with “$k - 1$ vertices”. One does not need the bijection $\pi \rightarrow T(\pi)$ to see this. Any binary tree with $k$ endpoints has $k - 1$ vertices with two successors.

• p. 27, line 7. Add “let” after “Now”.

• pp. 30 and 31, Figures 1-6 and 1-7. The shading of these figures that appeared in the original printing was omitted from the second printing. In Figure 1-6, the boxes are shaded to denote the Young diagrams of the partitions $\emptyset$, (1), (2), (1,1), (3), (2,1), (3,1), (2,2), (3,2), (3,3) in that order. In Figure 1-7, the boxes should be shaded above the lattice path $L$ so that the shaded boxes form the Young diagram of the partition (4,3,1).

• p. 33, Twelvelfold Way, entries 7 and 10. The sum for entry 7 should begin $S(n,0)$. Similarly the sum for entry 10 should begin $p_0(n)$. These terms are only relevant when $n = 0$ and yield the correct values $S(0,0) = p_0(0) = 1$.

• p. 34, line 9–. Change (24a) to (24b).

http://algo.inria.fr/flajolet/Publications/books.html

p. 45, Exercise 8(b). Add at the end of this exercise: “(Set \( \binom{m}{i} = 0 \) if \( i < 0 \).”

p. 47, Exercise 19(c), line 5. Change \( f(n) \) to \( f_k(n) \).

p. 50, Exercise 1.40. The statement of this exercise is somewhat misleading, since the solution gives a formula for \( a_i \) not in terms of the \( f_n \)’s, but rather in terms of the \( g_n \)’s defined by \( \log F(x) = \sum_{n \geq 1} g_nx^n \).

p. 52, Exercise 2(a), line 3. Change \( x + i + n + 1 \) to \( x + n + 1 \).

p. 55, Exercise 9(b). A simple combinatorial proof was given by the Cambridge Combinatorics and Coffee Club (December 1999).

p. 56, Figure 1-14. The next-to-last dot should be circled.

p. 59, Exercise 26, line 3. Change \( m_k(\mu) \) to \( f_k(\mu) \).

p. 59, Exercise 26, line 2–. Change function to functions.

p. 59, Exercise 26. The following historical remarks concerning this exercise may be of interest. I discovered the result in 1972 and submitted it to the Problems and Solutions section of the *Amer. Math. Monthly*. It was rejected with the comment “A bit on the easy side, and using only a standard argument.” My guess is that the editors did not understand the actual statement and solution of the problem. I had mentioned the result to Daniel I. A. Cohen, who included the case \( k = 1 \) as Problem 75 of Chapter 3 in his book *Basic Techniques of Combinatorial Theory*, Wiley, New York, 1978. For this reason the case \( k = 1 \) is sometimes called “Stanley’s theorem.” An independent proof of the general case was given by Kirdar and Skyrme, as mentioned in the text (page 59). The generalization from \( k = 1 \) to arbitrary \( k \) was independently found by Paul Elder in 1984, as reported by R. Honsberger, *Mathematical Gems III*, Mathematical Association of America, 1985 (page 8). For this reason the general case is sometimes called “Elder’s theorem.” A further proof was given by A. H. M. Hoare, *Amer. Math. Monthly* 93 (1986), 475–476.

p. 61, Exercise 33, line 1. Change \( A(n,k)2^k \) to \( A(n,k+1)2^k \), and change \( k \) at the end of the line to \( k - 1 \).
• p. 62, line 8–. Change \( \sum_{n \geq 0} \) to \( \sum_{i \geq 0} \).

• p. 62, line 1–. Change \( \mathfrak{S}_{2n+1} \) to \( \mathfrak{S}_{2n-1} \).


• p. 67, line 3. Insert a space before “has”.

• p. 70, equation (21). Change \( s_j \) to \( s_i \) (twice).

• p. 71, line 12–. Add the following sentence before this line (which is needed in the statement of Theorem 2.4.1).

Define the *rook polynomial* \( r_B(x) \) of the board \( B \) by

\[
r_B(x) = \sum_k r_k x^k.
\]

• p. 72, line 10. Change “If” to “It”.

• p. 75, Theorem 2.4.4, lines 4–5. Change “only \( s_1 \geq 0 \) (i.e., \( s_i < 0 \) for \( 2 \leq i \leq t \)) to \( s_1 = 0 \) and \( s_i < 0 \) for \( 2 \leq i \leq t \)”.

• p. 80, lines 14– and 16–. Change “positive” to “nonnegative”.

• p. 80, lines 7– and 11–. Change \( \tau' \) to \( \tilde{\tau} \).

• p. 81, Figure 2-1. Change \( \tau' \) to \( \tilde{\tau} \).

• p. 82, §2.7, line 7. Change \( v_{i+i} \) to \( v_{i+1} \).

• p. 84, lines 11–13. Change the sentence “Property (a) . . . obtained from \( L \)” to “Property (a) follows since the triple \((i, j, v)\) can be obtained from \( L^* \) by the same rule as it can be obtained from \( L \)”.

• p. 84, Example 2.7.2, line 7. Change \( \alpha_i \) and \( \delta_i \) to \( \alpha_j \) and \( \delta_j \) (twice).


• p. 88, equation (44). Change \( 1/n^5 \) to \( 2/n^5 \).
• p. 89, Exercise 11, line 3. Change “nents” to “nent”.

• p. 89, Exercise 11(b), line 5. Change $G$ to $\overline{G}$.

• p. 92, line 5. Change $A^a$ to $V^a_T$.

• p. 92, items e,f. Change $|T|$ to $|A_T|$.

• p. 93, Exercise 8(d). Change $f(n) + f(n + 1)$ to $f_2(n) + f_2(n + 1)$.

• p. 94, solution to Exercise 14, line 5. Change $k \leq -1$ to $k \geq -1$.

• p. 95, line 2. Change “regular” to “that every connected component is regular”.

• p. 96, line 3 after equation (1). Delete comma after $C$.


• p. 111, Example 3.5.3, line 6. Change second $bacde$ to $badce$.

• p. 111, Figure 3-24. The shading of this figure that appeared in the original printing was omitted from the second printing. The entire inside region should be shaded.

• p. 111, line 3–. Change “$m$ element” to “$m$-element”.

• p. 112, lines 7– to 6–. Change $e(J(m + n)) = e(m + 1 \times n + 1) = \binom{m+n}{n}$ to $e(m + n) = \binom{m+n}{n}$.

• p. 117, line 13. Change $T < 1$ to $T < \hat{1}$.

• p. 117, line 4–. After $\iff$ insert “$f(0) = g(0)$ and”.

• p. 120, line 10. Insert $\Delta$ after “collection”.

• p. 122, Figure 3-29. Shading is missing from $\Gamma_4$, $\Gamma_5$, and $\Gamma_6$. All two-dimensional regions (including the outside one) are 2-cells.

• p. 125, line 6. Change $\mathbb{C}$ to $K$.

• p. 125, line 9–. Change $\mathbb{C}$ to $K$.

• p. 125, line 8–. Change $y \leq 1$ to $y \leq \hat{1}$.

• p. 127, lines 2 and 6. Change $L$ to $L_n$.

• p. 127, Example 3.10.3, line 2. Change “Note” to “For the purpose of this example, we say”. If one wants to retain the more standard convention that the empty set spans $\{0\}$, then we need to enlarge $L_n(q)$ by adding $\emptyset$ below $\{0\}$.
• p. 128, line 14. The type of a set partition \( \pi \in \Pi_n \) has not been defined, though the definition should be clear in analogy to the type of a permutation. Namely, define type(\( \pi \)) = (a_1, \ldots, a_n) if \( \pi \) has \( a_i \) blocks of size \( i \).

• p. 128, line 16. Change \( = \) to \( \cong \).

• p. 131, line 6 of text. Change “rank \( i \)” to “of rank \( i \)”.

• p. 131, line 5–. Change \( \sigma : P \rightarrow [n] \) to \( \sigma : P \rightarrow n \).

• p. 133, line 13–. Change \( a_0 = \hat{0}, a_{s+1} = \hat{1} \) to \( a_0 = 0, a_{s+1} = n \).

• p. 136, line 6. Change \((-1)^n \Delta Z(P, m - 1)\) to \((-1)^{n-1} \Delta Z(P, m - 1)\).

• p. 137, line 7. Change \( Q \) to \( P - Q \).

• p. 139, Exercise 1.193, line 2. Change \( \gamma_n \) to \( \gamma(n) \).

• p. 142, line 14–. It should have been stated that \( \text{card}(x, y) \) is short for \( \text{card}([x, y]) \).

• p. 152, reference 12, line 2. Change “functions” to “function”.

• p. 153, Exercise 1a, line 1. Change “operation” to “relation”.

• p. 155, line 4. Insert “irreducible” before “connected”. (An irreducible poset is one that cannot be written in a nontrivial way as a direct product.)

• p. 156, Exercise 15(d). This exercise was solved by J. D. Farley and R. Klippenstine, Posets with the same number of order ideals of each cardinality, II, preprint dated November 30, 2004.

• p. 157, Exercise 22(e). It was shown by J. Farley, J. Combinatorial Theory (A) 90 (2000), 123–147, that the only nondecreasing cover functions (with \( f(0) \geq 1 \)) are \( f(n) = k \) and \( f(n) = n + k \) for \( k \geq 1 \). This confirms a conjecture of R. Stanley, Fibonacci Quart. 13 (1975), 215–232 (page 226).

• p. 160, line 1. Change second \( P \) to \( \bar{P} \).

• p. 160, Exercise 32, line 4. Insert \( \mu(x_k, \hat{1}) \) after \( \mu(x_{k-1}, x_k) \).

• p. 164, line 1. Change \( a_i \) to \( y \) (twice).

• p. 166, line 1–. Change \( X_1 \) to \( H_1 \) and \( X_\nu \) to \( H_\nu \).

• p. 167, Exercise 59a. Change the last sentence to: Show that if \( Z(P, m + 1) = \sum_{i \geq 1} a_i \binom{m-1}{i} \), then \( Z(Q_0, m + 1) = 1 + \sum_{i \geq 1} a_i m^i \).

• p. 170, Exercise 70(a), line 2. Change \( \beta(P, S) \) to \( \beta(P_n, S) \).
p. 174, Exercise 81a, lines 3–4. Change “define $f, g$” to “define $g, h$”.

p. 174, Exercise 81c, line 5. Change this line to
\[ 1 + t \sum_{n \geq 1} G_n(q, t)x^n/(n!) = \left[ 1 - t \sum_{n \geq 1} (1 - t)^{n-1}x^n/(n!) \right]^{-1}. \]

p. 174, Exercise 81c, line 2–. Change $(1 - t)/(e^{x(t-1)} - 1)$ to $(1 - t)/(e^{x(t-1)} - t)$.

p. 175, Exercise 7(b). Replace the first paragraph with the following: Suppose that $f : \text{Int}(P) \to \text{Int}(Q)$ is an isomorphism. Let $f([\hat{0}, \hat{0}]) = [s, s]$, where $\hat{0} \in P$ and $s \in Q$. Define $A$ to be the subposet of $Q$ of all elements $t \geq s$, and define $B$ to be all elements $t \leq s$. Check that $P \cong A \times B^*$, $Q \cong A \times B$.

p. 178, solution to Exercise 19a, lines 5–7. Interchange $f_k$ and $g_k$.

p. 183, solution to Exercise 30, line 5. Change $Q$ to $\bar{P}$ (under the summation sign).

p. 184, solution to Exercise 32, line 3. Insert $\mu(x_k, \hat{1})$ after $\mu(x_{k-1}, x_k)$.

p. 184, solution to Exercise 37b, line 1. Change “$f(x, s) = x$” to “$f(x, s) = \phi(x)$ (so $F(x, s) = x$)”.

p. 187, line 2. Remove $\prod_{i=1}^k$.

p. 187, line 3–. Change $x^{n - \dim W}$ to $x^{n - \dim W'}$.

p. 188, Exercise 46, second solution, lines 1–2. Change $a \neq \{1\}$ to $\emptyset \neq a$.

p. 190, line 1. Change $x_1^{a_1} \cdots x_n^{a_n}$ to $x_1^{a_1} \cdots x_n^{a_n}$.

p. 191, Exercise 50, line 4. Under the third summation symbol, change $x \in P_j$ to $y \in P_j$.

p. 191, Exercise 51, line 5. Change “voltage graphs” to “gain graphs”.


p. 193, line 1. Change “chains” to “multichains”.

page 192, line 2–. This formula should be:
\[ Z(P \oplus Q, m) = Z(P, m) + Z(Q, m) + \sum_{j=2}^{m-1} Z(P, j)Z(Q, m+1-j), \quad m \geq 2. \]
• p. 195, solution to Exercise 61b. Interchange \( p \) and \( p1 \) throughout.

• p. 196, line 16–. Change \( x_{\rho(x_i)} \) to \( t_{\rho(x_i)} \).

• p. 197, Exercise 69(d), lines 2– to 1–. Update this reference to Discrete Math. 79 (1990), 235–249.

• p. 197, Exercise 70(a). Change \( \beta(P, S) \) to \( \beta(P_n, S) \).

• p. 200, Exercise 80, line 4. Change \( 6n + 3 \) to \( 6n + 3 \), i.e., the 3 should not be italicized. Could this be the most nitpicking error of this errata?

• p. 206, first line of proof. Change \( R(x) \) to \( F(x) \).

• p. 216, line 5. Change 1.3.3 to 1.3.

• p. 223, line 17. Change \( P \) to \( C \).

• p. 224, line 3. Change “rank \( d = \dim C \)” to “rank \( d + 1 \) where \( d = \dim C \)”.

• p. 227, line 8. Change “\( a_i = \lceil b_i \rceil \), the least integer \( \geq b_i \)” to “\( a_i = \lfloor b_i - 1 \rfloor \), where \( \lfloor b_i \rfloor \) denotes the least integer \( \geq b_i \)”.

• p. 230, line 2 of Proof. Change \( d - \dim \sigma \) to \( d - \dim \sigma + 1 \).

• p. 231, line 3. Change the initial minus sign to +.

• p. 231, Lemma 4.6.17(i). As stated, the result is false. For instance, let \( E = \mathbb{N} \) and \( a_1 = -1 \). Then \( G(\lambda) = 1 \) but \( E(\lambda^{-1}) = \sum_{n \geq 0} \lambda^{-n} \). We need to assume also that \( g(r) > 0 \) for at least one \( r > 0 \). We claim that then \( g(s) = 0 \) for all \( s < 0 \), and hence (i) follows. Let \( \alpha \in E \) satisfy \( L(\alpha) = r > 0 \), and suppose that there exists \( \beta \in E \) with \( L(\beta) = s < 0 \). Then for all \( t \in \mathbb{N} \) the vectors \( -ts\alpha + tr\beta \) are distinct elements of \( E \), contradicting \( g(0) < \infty \).

• p. 236, line 6. Change “union” to “intersection”.

• p. 232, line 6 (excluding the heading). Change “matrix” to “\( \mathbb{N} \)-matrix”.

• p. 233, line 9. Insert \( n \times n \) before \( \mathbb{P} \)-matrices.

• p. 233, lines 6– and 5–. A subscript \( n \) is missing from \( H \) four times.

• p. 235, lines 2– and 1–. Change \( \mathcal{P} \) to \( \mathcal{P}_m \).

• p. 237, line 8. Change \( \text{den}(\gamma, t) \) to \( \text{den}(\gamma/t) \).

• p. 240, Example 4.6.32(b), line 2. Change the coefficient of \( \bar{i}(\mathcal{P}, 1) \) from \( -1 \) to 1.
• p. 241, line 6. Change $a_k$ to $a_p$.

• p. 244, Figure 4-13. Add an edge from 13 to 31.

• p. 246, lines 5–6. Change 7 to 6 (twice).

• p. 249, line 15–. Add at end of line: where $b(n) = \sum_{v \in B_n} w(v)$.

• p. 253, line 11–. Change $i$-th to $k$th.

• p. 253, line 4–. Change $D_4$ to $D_3$.

• p. 256, line 2. Change $(j - 1, j)$ to $(j, j - 1)$.

• p. 256, line 10. Change $-$ to $+$.

• p. 257, line 8–. Change $f_{ij}(n_{j+1})$ to $f_{ij}(n_j)$.

• p. 260, Figure 4-42. Change the label on the edge from 00 to 01 from $F_1 \ast F_2$ to $F_1 \ast F_3$.

• p. 262, line 5–. The first published statement for the generating function for $F(x)$ appearing before equation (47) seems to be due to H. N. V. Temperley, Phys. Rev. (2) 103 (1956), 1–16.

• p. 262, line 2–. The result of Hickerson has now appeared in J. Integer Sequences (electronic) 2 (1999), Article 99.1.8,

  \texttt{http://www.research.att.com/~njas/sequences/JIS/HICK2/chcp.html}.

• p. 265, line 2–. Change $\mathbb{Z}/n\mathbb{Z}$ to $(\mathbb{Z}/n\mathbb{Z})^2$.

• p. 266, Exercise 12. Change $\Phi \alpha = 0$ to $\Phi \alpha = \beta$, where $\beta$ is a vector of linear polynomials $an + b$. Moreover, the final sentence should be “Show that $\mathbb{N}^m$ can be partitioned into finitely many regions such that on each region the number of solutions is a quasipolynomial in $n$ for $n$ sufficiently large.” It is possible that this result is already known.

• p. 271, Exercise 27(a), line 2. Change this line to

  \[ x_1 + x_2 + \cdots + x_r \leq 1, \ y_1 + y_2 + \cdots + y_s \leq 1, \ x_i \geq 0, \ y_i \geq 0. \]

• p. 271, Exercise 28(d). We must also define $i(Q, 0) = 1$, despite the fact that the value of the polynomial $i(Q, n)$ at $n = 0$ is $\chi(Q)$, the Euler characteristic of $Q$. 

8
• p. 277. Exercise 5, line 2. Change $F(0) \neq 0$ to $G(0) \neq 0, \infty$.

• p. 280, Exercise 14(a), first line of ii. Change $V_i$ to $v_i$.

• p. 281, line 11. Change $x^y$ to $x^\gamma$.

• p. 285, Exercise 23. Peter McNamara has pointed out a gap in the proof. Namely, from the fact that $P - M$ and $P - M'$ are disjoint unions of chains, it need not follow (when $P$ has maximal chains of length one) that $P$ is a disjoint union of chains, together with the stated relations $x < y$. To fix the proof, note that $m$ is the largest power of $x_0$ that can appear in a monomial in $\bar{G}(P, x)$. Hence $m$ is the largest power of any $x_i$ that can appear in a monomial in $\bar{G}(P, x)$. Let $A$ be an antichain of $P$. We can easily find a strict $P$-partition that is constant on $A$, so $\#A \leq m$. Hence the largest antichain of $P$ has size $m$. By Dilworth’s theorem, $P$ is a union of $m$ chains. Each such chain intersects $M$ and $M'$. It is now easy to see that if $P - M$ and $P - M'$ are disjoint unions of chains, then indeed $P$ is a disjoint union of chains together with the stated relations $x < y$, and the proof proceeds as before.

• p. 290, Exercise 34(a), line 1. Change $(1, \zeta, \zeta^2, \ldots, \zeta^{n-1})^t$ to $(1, \zeta^r, \zeta^2r, \ldots, \zeta^{(n-1)r})^t$.

• p. 293, line 9–. In the definition of a connected graph, it should also be specified that the empty graph is not connected.

• p. 294, line 5. Change “define” to “defining”.

• p. 294, line 8. Insert a comma after $\{1, 3, 4, 5, 8, 9\}$.

• p. 294, line 8–. Change “defined” to “define”.

• p. 295, Figure A-2. Interchange the labels 3 and 5 on the third tree.


• p. 310, Problem 25, line 2. Change “occurences” to “occurrences” (twice).

• p. 310, Problem 25, line 3. Change “$t$ occurences of each $a_{ij}$” to “$2t$ occurrences of each $a_{ij}$”.

• p. 314, Problem 6(c). A solution was found by Ethan Fenn (private communication, November, 2002). The rating should be changed to [3–] and the problem restated as follows.

Let $k \geq 3$, and let $P_k$ denote the poset of all subsets of $\mathbb{P}$ whose elements have sum divisible by $k$. Given $T \leq S$ in $P_k$, let

$$i_j = \# \{ n \in T - S : n \equiv j \pmod{k} \}.$$
Clearly $\mu(S, T)$ depends only on the $k$-tuple $(i_0, i_1, \ldots, i_{k-1})$, so write $\mu(i_0, \ldots, i_{k-1})$ for $\mu(S, T)$. Show that

$$\sum_{i_0, \ldots, i_{k-1} \geq 0} \mu(i_0, \ldots, i_{k-1}) \frac{x_0^{i_0} \cdots x_{k-1}^{i_{k-1}}}{i_0! \cdots i_{k-1}!} = k \left[ \sum_{j=0}^{k-1} \exp \left( x_0 + \zeta^j x_1 + \zeta^{2j} x_2 + \cdots + \zeta^{(k-1)j} x_{k-1} \right) \right]^{-1},$$

where $\zeta$ is a primitive $k$th root of unity.

- p. 310, Problem 28, line 4. Change $L_{n+1}$ to $L_n$.
- p. 314, Problem 13, lines 2–3. Delete the difficulty rating [2+] at the beginning of these lines.
- p. 314, line 2–. Delete “(ii)”.
- p. 318, Problem 10, line 3. Change $H(a)$ to $H_n(a)$.
- p. 318, Problem 11(b), line 1. Insert “the” after “with”.
- p. 318, Problem 12(b), line 2. Change $\sum_{\alpha \in \Phi_3}$ to $\sum_{\alpha \in E_{\Phi_3}}$.
- p. 318, Problem 13. This should be rated [2+].
- p. 319, line 9–. This erratum is unnecessary and can be deleted.
- p. 320, line 2 (paperback edition only). Change $(2^{a_1} - 1) \cdots (2^{a_1} - 1)$ to $(2^{a_1 - 1} - 1) \cdots (2^{a_k - 1} - 1)$.
- p. 320, lines 16–17. Delete this item.
- p. 321, item 3, line 2. Change $\mu(kn)$ to $\mu_S(kn)$.
- p. 321, line 7. Delete this entry (for p. 149, line 10).
• p. 322, lines 8– to 5–. The stated result is false. The hypothesis on $L$ should be the following: $L$ is an $n$-element lattice such that for all $0 < x \leq y$ in $L$, there exists $z \neq y$ such that $z \lor x = y$.

• p. 322, line 4–. Change $\ell^{k-1}$ to $k^{\ell-1}$ ($\ell \geq 2$).

• p. 324, line 9–. Change 22 to 29.